CHAPTER 1.9

Viscosity and Rheology

Michael G. Nelson

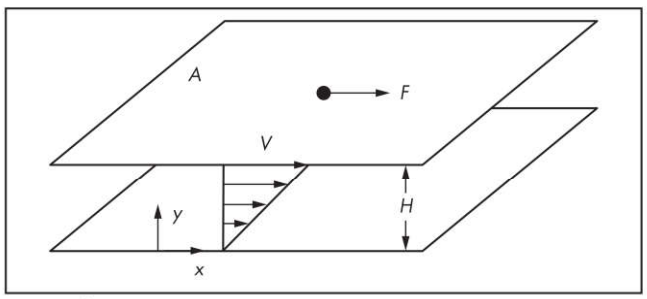
Operations in mineral processing and hydrometallurgy frequently include the handling of fluids—gases, liquids, and slurries. A fluid is a substance that undergoes continuous deformation, or strain, when subjected to shear stress; the relation of shear stress and shear strain in a fluid is characterized by the fluid's viscosity and rheology.

Cowper (1992) provides an excellent description of the importance of fluid viscosity in mineral processing and hydrometallurgical operations. Cowper notes the following effects:

- Grinding is most efficient at viscosities of 100–300 mPa·s, and coal grinding mill throughputs fall off rapidly when the slurry yield stress exceeds 10–20 Pa. Typical gold ore mills operate with slurry yield stress of 5–7 Pa, while cement plant ball mills run at 20–40 Pa.
- Classification is much less efficient at high viscosity, which implies high solids content.
- Flow through screen openings is generally laminar. For a Newtonian fluid at a given head, screen throughput is approximately inversely proportional to the viscosity of the feed. For non-Newtonian fluids, throughput depends mainly on the slurry yield stress, and the dependence is nonlinear.
- In hydrocyclones, when the slurry yield stress is too low, the cut point decreases and undersized particles may be unnecessarily returned to the mill, with a consequent reduction in throughput and increase in circulating load. On the other hand, if the yield stress is too high, oversized particles may pass through the cyclone for processing, with values unliberated.
- Within practical constraints, leach tanks should be operated at the highest possible slurry density, increasing retention time for a given solids throughput. However, high pulp densities can hinder carbon-in-pulp leaching and adsorption kinetics, so a balanced approach is appropriate.
- The pumping of thickener underflows—concentrates and tailings—is very sensitive to slurry rheology. A small change in solids concentration can cause a significant change in slurry yield stress. In normal operation, the underflow pipeline operates at a constant flow and a relatively constant head in turbulent flow. If the solids concentration in the slurry increases, the flow may transition to the laminar region, and the system head will increase, and if the system head exceeds the pump capacity, flow will stop. Similarly, on the suction side of the pump, an increase in solids concentration may reduce the available suction head of the pump. Because most centrifugal slurry pumps do not provide sufficient net positive suction head to prevent cavitation, the thickener outlet may become plugged with a high-density, high-viscosity slurry, effectively shutting down the thickener.

VISCOSITY AND RHEOLOGY

Figure 1 shows the behavior of a fluid experiencing a simple shear stress, which results in a simple shear flow. The fluid is between two large parallel plates of area A, separated by a small distance H. The bottom plate is fixed and a force F acts on the upper plate, causing it to move at a velocity V. The fluid will continue to deform as long as the force is applied, unlike a solid, which will experience a finite deformation and then fail. The shear stress in this illustration is $\tau = F/A$. Under a no-slip condition, the fluid at the lower plate will have a velocity of zero, and the fluid at the upper plate will move with the upper plate at velocity V. This will result in a velocity gradient in the fluid between the plates, given by $v_y = Vy/H$. The velocity gradient, dv/dy, is defined as the shear rate and has units of s^{-1}.



Source: Tilton 2007

Figure 1 Movement of a fluid under simple shear stress

Michael G. Nelson, Professor & Chair, Mining Engineering Department, University of Utah, Salt Lake City, Utah, USA

Dynamic viscosity is simply defined as the ratio of shear stress to shear rate, thus

$$\mu = \tau/(dv/dt) \quad \textbf{(EQ 1)}$$

In the metric system, viscosity has units of kilograms per meter-second (kg/m·s), Newton-seconds per square meter ($N \cdot s/m^2$), or pascal-seconds (Pa·s). It is also expressed in poise (P) or centipoise (cP), where 1 Pa·s = 10 P or 1,000 cP. These are the standard units used for viscosity. The viscosity defined by Equation 1 is also known as the absolute or shear viscosity. The kinematic viscosity (υ) is the ratio of viscosity (μ) to density (ρ) given by

$$\upsilon = \mu/\rho \quad \textbf{(EQ 2)}$$

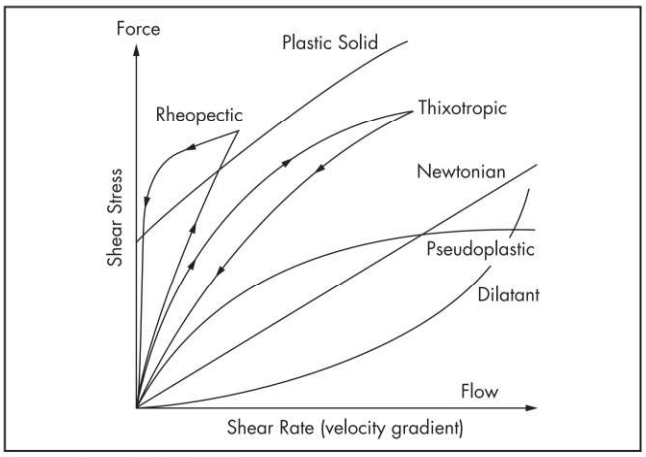
and has units of m^2/s or stokes (St), where 1 St = 1 cm^2/s, and 1 cSt (centistoke) = 1 mm^2/s.

Experience shows that fluid movement is often more complex than that just described. *Rheology* is the study of the relationship between fluid deformation and stress. Most solids exhibit some degree of elasticity, in which they show reverse deformation when stress is removed. Fluids that do not exhibit any solid-like, elastic behavior are called *purely viscous fluids*, while those that exhibit viscous and elastic behavior are classified as *viscoelastic*.

The behavior of purely viscous fluids may be either time independent or time dependent. In the first case, the shear stress depends only on the instantaneous shear rate, while in the second case the shear stress also depends of the rate of deformation in the past, as the result of changes in structure or orientation during shear deformation.

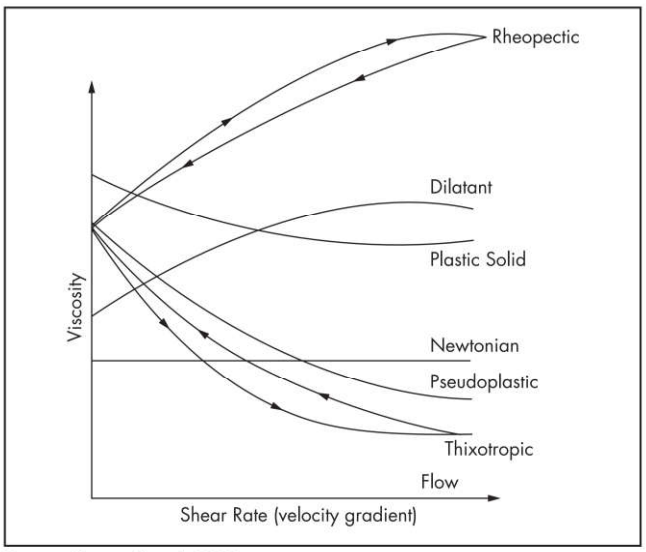
Fluid rheology is usually characterized by a rheogram for the fluid. A rheogram is a plot of the shear stress versus the shear rate for a fluid in simple shear flow. Figure 2 shows rheograms for six types of fluids, as classified by rheology:

1. The rheogram for a Newtonian fluid is linear and intersects the origin of the graph. Newtonian fluids are the most common and include gases and liquids of low molecular weight such as water, solutions of salts in water, gasoline, kerosene, and light oils.
2. Pseudoplastics are shear-thinning materials whose apparent viscosity drops as flow or shear rate increases. Some substances exhibit a yield stress above which the apparent viscosity drops, so that a unit increase of driving force results in more and more flow. Pseudoplastic materials include catsup, paper pulp, and printer's ink.
3. Dilatant materials show shear thickening. Their apparent viscosity increases as the flow increases, and more and more stress is required to obtain the same increase in flow. A mixture of cornstarch and water is dilatant; other dilatant materials include quicksand, peanut butter, and many candy compounds.
4. Plastic solids are true plastics in the sense that they normally behave like solids, but when the shear stress reaches their yield point, they behave as viscous fluids and start to "cold flow." Most plastics, chewing gum, tar, and some oils exhibit this behavior.
5. Thixotropic materials are usually pseudoplastic, shear-thinning substances, but they exhibit hysteresis. For example, a thixotropic material will require less power when re-agitated than was required during the first agitation. Thixotropic substances include asphalt, lard, silica gel, most paints, glues, and fruit juice concentrates.



Source: Kim and Liptak 2003a

Figure 2 Rheograms for various types of fluids



Source: Kim and Liptak 2003a

Figure 3 Variation of viscosity with shear rate for various types of fluids

6. Rheopectic substances also display hysteresis, but instead of a shear thinning, they thicken with increasing shear. Their viscosity appears to increase, and some will "set" after a period of agitation. Gypsum in water is an example of this behavior.

Non-Newtonian fluids are those in which the viscosity varies with the shear rate. For these fluids, the viscosity is usually referred to as the *apparent viscosity*, to emphasize their distinction from Newtonian fluids. Figure 3 shows how viscosity varies with shear rate for the six types of fluids previously described.

LABORATORY VISCOMETERS AND RHEOMETERS

All instruments for measuring viscosity and rheology in the laboratory are based on one of two principles—measuring the movement or flow of the fluid in a system with a standard configuration, or measuring the movement of a standard object or mechanism in the fluid. Some laboratory instruments are

designed solely for use in the lab and some in the lab and the plant.

Bubble-Time Viscometer

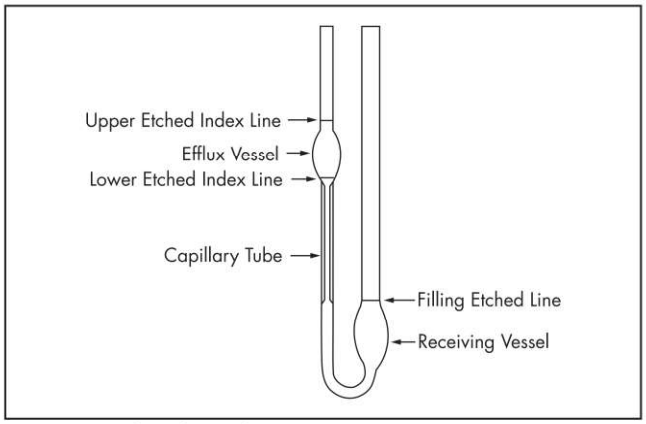
In the bubble-time viscometer, a liquid flows down in the annular zone between the wall of a sealed tube and the perimeter of a rising air bubble. The kinematic viscosity of the fluid is determined by comparing the rising velocity of the bubble in the subject fluid with the bubble velocity in a tube of the same size filled with a fluid of known viscosity. This method is easy to use, and the equipment requires no calibration. While it is usually used in a laboratory, it can be used in an operational setting if a sample of the test fluid can be drawn. It is suitable only for Newtonian fluids.

Capillary Viscometers

The manual capillary viscometer measures the time required for the sample fluid to flow through a capillary tube. The Ostwald viscometer, shown in Figure 4, is commonly used. A fixed volume of the sample fluid is introduced to the lower receiving vessel, and the vessel is placed in a constant-temperature bath. After the sample temperature has stabilized (usually about 5 minutes), the sample is sucked up into the efflux vessel until the fluid level is above the upper index line. Suction is released so the fluid flows down through the capillary, and the time of flow between the upper and lower index lines is recorded. The flow time is multiplied by the viscometer's calibration constant to obtain the kinematic viscosity. The method is limited in application to Newtonian fluids. Because the measurement is conducted under atmospheric conditions and given that it takes several minutes, it should not be used with volatile or atmospherically unstable fluids. The sample fluid must also be free of solids. The method is used in both laboratory and industrial applications. Automatic capillary viscometers are available and can be used for online process control when coupled with an automatic sampling system.

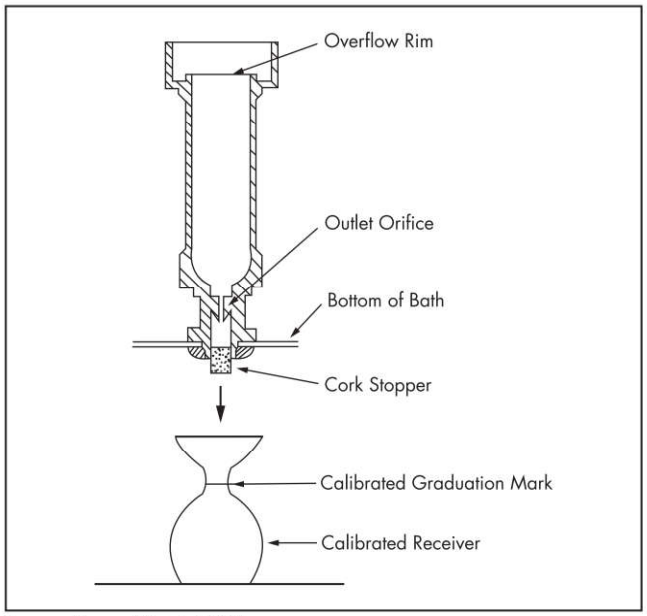
Capillary-Extrusion Viscometers

Capillary-extrusion viscometers were designed to overcome the limitations of capillary-tube viscometers, which can measure only low-viscosity, Newtonian fluids. These instruments use a controlled pneumatic or hydraulic pressure system to force the sample fluid through an orifice. The kinematic viscosity is determined from the measured flow rate through the orifice and the driving pressure. These instruments are used only in the laboratory, and care must be taken to correct for errors that result from nonuniformity in the sample shear rate, entrance and exit effects at the orifice, fluid compressibility, loss of pressure resulting from flow in the sample chamber, and uneven temperatures produced by shear-induced heating.



Source: Kim and Liptak 2003b

Figure 4 Ostwald manual capillary viscometer



Source: Kim and Liptak 2003b

Figure 5 Saybolt viscometer

Efflux-Cup Viscometers

Efflux-cup viscometers are widely used in the field for measuring the viscosity of oils, syrups, paints and varnishes, and bitumen emulsions. The governing principles and operating procedures are similar to those for capillary-tube viscometers. The user measures the time required for a known volume of the sample fluid to flow through a fixed orifice at the bottom of a cup and into a receiving vessel. The efflux time is converted to kinematic viscosity using a conversion chart or formula. There are many types of efflux-cup viscometers. The most common is the Saybolt viscometer, shown in Figure 5. It is commonly used for testing petroleum products. Viscosity measured with a Saybolt viscometer is expressed directly in Saybolt universal seconds, or SUS—the time required for 60 cm^3 of sample to flow through an orifice with a diameter of 0.176 cm and a length of 1.225 cm. Saybolt universal seconds, t, can be converted to kinematic viscosity, υ, using one of the following equations:

$$\upsilon = 0.226\,t - 195/t \text{ cSt, when } t < 100 \text{ s} \quad \textbf{(EQ 3a)}$$

$$\upsilon = 0.220\,t - 135/t \text{ cSt, when } t > 100 \text{ s} \quad \textbf{(EQ 3b)}$$

Ford-cup and Zahn-cup viscometers are used in measuring low-viscosity liquids, primarily paints, varnishes, and other finishes. Automated Zahn-cup viscometers are used for online measurements in industrial applications.

Falling-Ball Viscometers

The falling-ball viscometer uses a precision-bored glass tube, usually about 200 mm long and 16 mm inside diameter, fitted

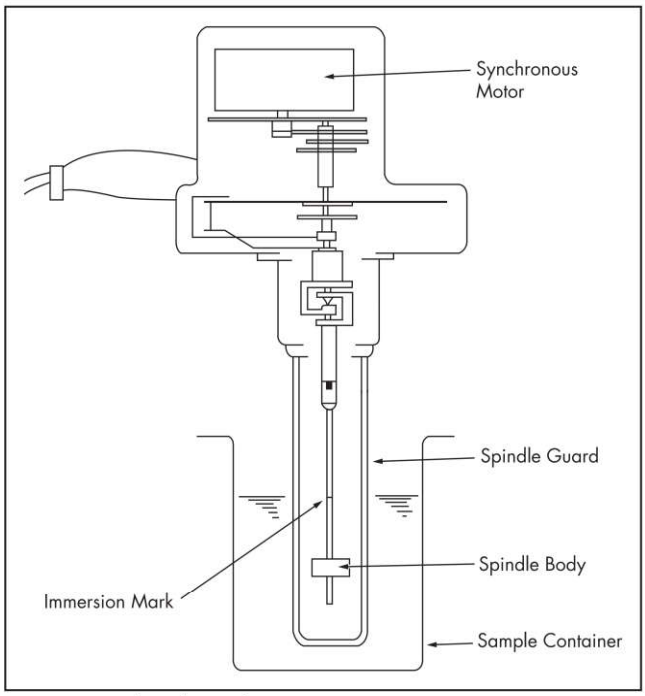
with a capillary plug and an assortment of calibrated glass or steel balls. The viscometer is enclosed in a water jacket, so temperature can be held constant during tests. Measurement is made by timing the fall of the ball through an accurately calibrated distance, and absolute viscosity is calculated as a function of the fall time, the specific gravities of the ball and the fluid at the measuring temperature, and a ball constant, based on Stokes' law. In the range of 0.01 to 0.6 Pa·s, experienced personnel can determine absolute viscosity with an accuracy of ±0.1%. Measurements of this accuracy also require a viscometer that is clean and well maintained, a stopwatch with an accuracy of 0.02 seconds, and careful control of temperature during the measurements.

Falling-needle viscometers are similar to falling-ball devices but use a needle to provide a more stable falling motion and minimize wall effects. Automated viscometers of both types are available.

Rotational Viscometers

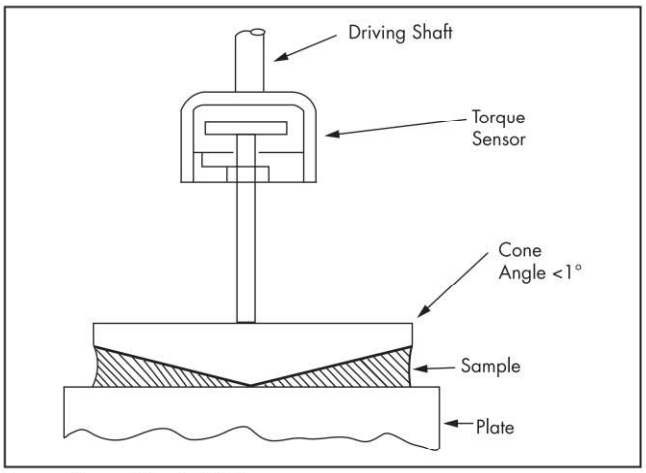
Rotational viscometers operate on the principle that the torque necessary to overcome the viscous resistance to the movement induced by the rotation of a spindle is directly proportional to the viscosity of the fluid. The entire sample is subjected to a uniform or near-uniform shear rate, and viscosity is determined directly by measuring the corresponding shear stress. A properly equipped rotational viscometer can measure fluid viscosities ranging from 10^{-5} to 10^{7} Pa·s at a range of shear rates from 10^{-4} to 10^{4} s^{-1}. Rotational viscometers can perform continuous measurements under varying conditions on the same sample and can also operate at a given set of boundary conditions for an extended time period.

Figure 6 illustrates a rotational viscometer. All rotational viscometers have a mechanism for driving a spindle at a constant speed and a torque-measuring device. Shear rate



Source: Kim and Liptak 2003b

Figure 6 Simple rotational viscometer



Source: Kim and Liptak 2003b

Figure 7 Cone-and-plate viscometer

is determined as a function of spindle diameter and speed of rotation. There are several design variations. Some involve the rotation of a disc or spindle in the fluid; others rotate the container around the fluid and measure the torque required to restrain rotation of the suspended spindle.

Spindle-type rotational viscometers work well with Newtonian fluids, but errors can arise when they are used with non-Newtonian materials, because the property that is being measured—viscosity—is affected by the shear applied to it. Further inaccuracies can arise from the end effects around the disc and from heat generated at high shear rates. Alternative designs can overcome these errors. One approach is to use two coaxial cylinders; another is the cone-and-plate viscometer, shown in Figure 7. At a constant angular velocity, the cone-and-plate configuration provides a uniform shear rate and stress throughout the fluid sample, and heating effects are reduced because the sample is contained in a relatively thin layer.

Straight-Pipe Viscometers

Using the following Hagen–Poiseuille equation, the kinematic viscosity μ of a fluid flowing through a pipe section of length L can be calculated as

$$\nu = \Delta P \pi R^4 / 8LQ \qquad \textbf{(EQ 4)}$$

where

ν = kinematic viscosity of the fluid
ΔP = pressure drop over length L
R = radius of the pipe
Q = volumetric flow rate

This relationship provides a useful way to determine the viscosity of a fluid, literally *inline*, using only a straight pipe, a flowmeter, and a differential pressure sensor. It is used in the petroleum, chemical, and food processing industries. Unfortunately, the Hagen–Poiseuille equation applies only to laminar flow of Newtonian fluids. Application to other conditions requires approximations based on constitutive relationships. A method described by Hollinderbäumer and Mez (1998) uses the yield stress and the shear rate to calculate the *consistency* of a non-Newtonian fluid. The consistency, η, is

comparable to the dynamic viscosity of a Newtonian fluid and is given by

$$\eta = \tau - \tau_o + \gamma^* \quad \text{(EQ 5)}$$

where

τ = shear stress, Pa
τ_o = yield stress, Pa
γ^* = shear rate, s^{-1}

The method also includes an empirical structure exponent, *n*, an integer selected to account for the pseudo-plastic behavior of slurries. The resulting equation, shown below, provides a relationship among flow velocity, pressure drop, and consistency, but it is rather unwieldy and has not been widely used:

$$w_m = \frac{1}{R^2} \cdot \frac{1}{\frac{1}{n}+1} \cdot \left(\frac{1}{2} \cdot \frac{1}{\eta} \cdot \frac{\Delta p}{\Delta l}\right)^{\frac{1}{n}} \cdot \left[R^2 \cdot (R - r_o)^{\frac{1}{n}+1} - 2 \cdot \frac{(R - r_o)^{\frac{1}{n}+3}}{\frac{1}{n}+3} - 2 \cdot r_o \frac{(R - r_o)^{\frac{1}{n}+2}}{\frac{1}{n}+2} \right] \quad \text{(EQ 6)}$$

where

w_m = linear velocity, m/s
R = pipe radius, m
$r_o = 2\tau_0/(\Delta P/L)$
$\Delta p/\Delta l = \Delta P/L$, Pa/m, from Equation 4

INDUSTRIAL VISCOMETERS AND RHEOMETERS

In industrial or process applications, there are two types of viscosity measurement, *behavioral* and *analytical*. A behavioral measurement determines the viscosity of the fluid at the process temperature; an analytical measurement includes the viscosity at one or more reference temperatures. In many applications, a behavioral measurement is sufficient. This is the case in mineral processing—the behavior of the fluids is the primary concern. When the qualities or properties of a fluid must be controlled, an analytical viscosity measurement may be required. For example, when blending lubricants, the viscosities of the components at both 40°C and 100°C are required.

Industrial viscometers are understood to be those that can be installed in a tank or a pipe and operated continuously as process control instruments. Eight types of industrial viscometers are described in detail by Kim et al. (2003): continuous capillary, Coriolis, falling-element, float, oscillating, plastometer, rotational, and vibrational. Continuous capillary instruments are suitable only for Newtonian fluids; plastometers are used solely to characterize the molecular weight distribution of polymers and the plastic behavior (melt flow) of plastic materials. The other six types can be used to measure non-Newtonian fluid viscosities, but each has limitations. Of the standard process viscometers, Coriolis meters are the best suited to use with mineral slurries. Hollinderbäumer and Mez (1998) propose the use of a straight-pipe viscometer for controlling the flow of paste backfill.

Four types of commercially available viscometers are commonly used in measuring viscosity of slurries.

1. Falling-element instruments are not recommended for shear-sensitive fluids, nor for slurries with particles larger than about 0.5 mm or high solids content. They require a sample extraction system for online use and frequent cleaning when used with slurries.
2. Float viscometers will be plugged by particles larger than about 2.5 mm. They also require a sample extraction system for online use, frequent cleaning when used with slurries, and constant flow rate and pressure.
3. Oscillating, rotational, and vibrational viscometers can be installed directly in a pipe or tank and are not susceptible to plugging by large particles. However, in each of these instruments, the sensing element is immersed in the fluid and is therefore subject to wear from flowing fluid, especially if the fluid is a slurry.
4. Coriolis meters were originally designed to measure fluid flow, by sensing the torsional movement of a specially designed and instrumented pipe section. Because these meters have no internal components, problems with plugging and wear are minimized. However, because of the precision design and construction of the meters, they are available only in sizes up to 80 mm inside diameter.

The following discussion is limited to instruments that are suitable for measuring the viscosity of slurries, as that is the most common need in mineral processing. Other process instruments are discussed in detail in Kim et al. (2003). Most of the instruments used for measuring slurry viscosity function by measuring a stream of slurry diverted from the main process line. Some of these instruments can function continuously, while others require periodic washout or flushing.

Coriolis Mass Flowmeters

Coriolis mass flowmeters are widely used in the process industries. They are capable of measuring mass flow, density, temperature, and viscosity. They have no moving parts and can be installed directly in a process flow line. Coriolis meters are provided by several vendors with variable designs. The design used by Endress+Hauser (2018) is discussed here as an example of the measuring principle, which is based on the controlled generation of Coriolis forces. These forces are always present when both translational and rotational movements are superimposed, and are described by the following equation:

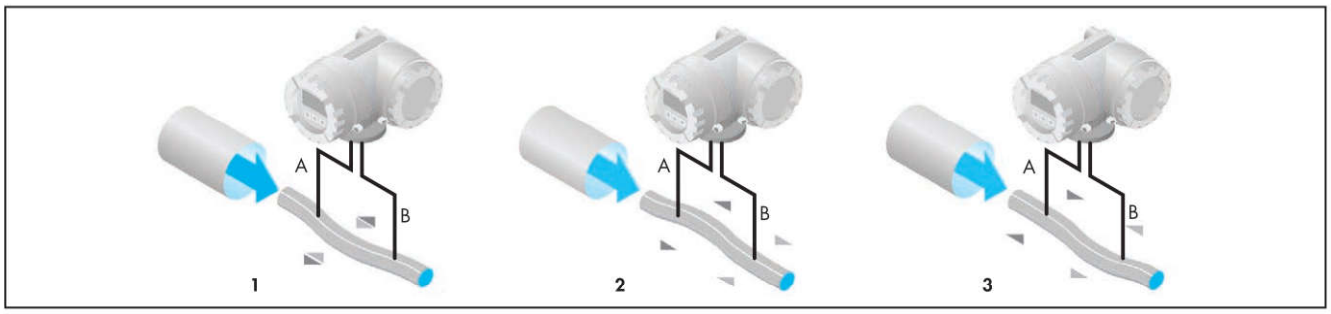
$$FC = 2\Delta m(v\omega) \quad \text{(EQ 7)}$$

where

FC = Coriolis force
Δm = moving mass
v = velocity of the moving mass in a rotating or oscillating system
ω = rotational velocity

The amplitude of the Coriolis force depends on the moving mass Δm, its velocity v in the system, and thus on the mass flow. Instead of a constant angular velocity ω, the Endress+Hauser sensor uses induced oscillation of the tube through which the fluid flows. The Coriolis forces produced at the measuring tubes cause a phase shift in the tube oscillations.

Referring to Figure 8, when there is no flow in the pipe, the oscillation measured at points A and B has the same phase, so there is no phase difference (1). Mass flow causes deceleration of the oscillation at the inlet of the tubes (2) and



Courtesy of Endress+Hauser

Figure 8 Operation of the Endress+Hauser Proline Promass 80I and 83I mass flowmeters

acceleration at the outlet (3). The phase difference (A-B) increases with increasing mass flow. Electrodynamic sensors register the tube oscillations at the inlet and outlet. The measuring tube is continuously excited at its resonance frequency. A change in the mass and thus the density of the oscillating system (comprising the measuring tube and fluid) results in a corresponding, automatic adjustment in the oscillation frequency. The resonance frequency is thus a function of fluid density. The microprocessor utilizes this relationship to obtain a density signal. The measuring principle operates independently of temperature, pressure, viscosity, conductivity, and flow profile.

The use of Coriolis mass flowmeters for determination of viscosity was described by Kalotay and Van Cleve (1994) and expanded by Van Cleve and Loving (1997). The oscillation of the tube also creates a velocity profile in the cross section of the tube. Viscosity measurement is based on the shape of that profile, on the shear forces, and on the corresponding shear rates developed in the fluid. The shear forces dampen the tube oscillation, so a larger driving force is required to maintain that oscillation, and viscosity can be measured as a function of the excitation current for tube oscillation and torsional movement. Coriolis mass flowmeters are available in standard pipe sizes up to 80 mm. Coriolis meters can be used to measure viscosities up to 500 cSt, depending on the diameter of the meter, with accuracy of ±1% of the span. Recent application of Coriolis meters for viscosity measurement in South Africa is described by Drahm and Matt (2004).

Pressure Vessel Rheometer

The rheometer described by Bakshi et al. (1999) extracts a sample from a process line. An absolute pressure transducer measures the pressure of the air inside the vessel, and a differential pressure transducer measures the pressure difference across the tube. The flow rate is calculated based on the pressure change in the vessel, and the measurements over time are used in the Hagen–Pouiselle equation (Eq. 4) to calculate viscosity. Correlation of measurement made with this instrument showed excellent correlation with measurements made using standard bench-top viscometers. This instrument must be drained after each measurement, but the filling and draining cycle can be automated.

Reeves Online Viscometer

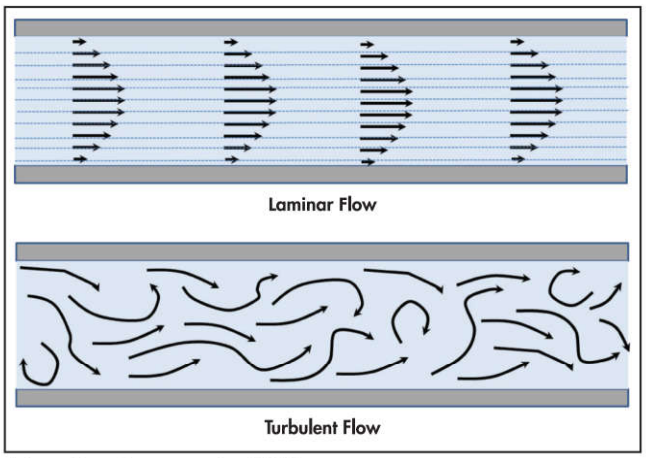
Napier-Munn et al. (1989) describe an online viscometer developed by Reeves (1985). Slurry drawn from the process line is pumped to a steady head box, where tramp oversize is removed, before flowing under gravity to a rotating bobbin. Current drawn by the bobbin motor is a function of drag on the bobbin and rotation rate, which can be related to viscosity using a simple calibration table prepared using fluids of known viscosity.

Slump Plate

Although not really a viscometer, this portable unit gives a quick visual indication of whether a slurry is likely to be pumpable with a centrifugal pump. It consists of a 300-mm^2 by 2-mm-thick stainless, flat plate with concentric rings inscribed on one surface. The central ring is 50 mm in diameter and the other diameters are progressively larger by 20 mm each. A short, thin pipe, 50 mm long with a 50-mm inside diameter and having machined and squared ends, is placed in the middle of the plate and a sample of slurry is poured into it. The pipe is then gently lifted vertically off the plate, allowing the slurry to spread or slump out evenly all around. If the slurry does not spread out to at least the third ring, it is too thick and a centrifugal pump will probably not be able to pump it.

VISCOSITY AND FLUID FLOW

Fluid flow can be laminar or turbulent. In laminar flow, the fluid velocity components vary smoothly with position, and with time if the flow is unsteady and smooth streamlines exist. The fluid flows in parallel layers, with no disruption between the layers. In turbulent flow, there are no smooth streamlines, and the velocity shows chaotic fluctuations in time and space. Figure 9 shows these two types of flow in a pipe cross section.



Adapted from Joseasorrentino 2014

Figure 9 Laminar and turbulent flow

In laminar flow, the energy input to the fluid functions to move the fluid in the direction of flow, to overcome friction in the flowing fluid and between the fluid and the wall of the pipe. In turbulent flow, some energy is dissipated in fluid turbulence, as indicated by the small, multi-directional arrows in Figure 9.

The nature of flow in a pipe depends on the pipe diameter, the density and viscosity of the fluid, and the flow velocity. These quantities can be combined into a single, dimensionless number called the Reynolds number, N_{Re}. It is one of several dimensionless numbers used to characterize dynamic similitude in a hydraulic system. Others are the Euler, Froude, Cauchy, and Weber numbers. Each number is the ratio of inertia to a particular force that may affect fluid flow. The Reynolds number, N_{Re}, is the ratio of inertia to viscous force, expressed in fundamental quantities as

$$N_{Re} = Ma/\tau A \quad \textbf{(EQ 8)}$$

where

M = mass
a = acceleration
τ = shearing stress
A = area over which the shearing stress acts

For fluids flowing in a pipe, the Reynolds number is expressed as

$$N_{Re} = DV/\upsilon \quad \textbf{(EQ 9)}$$

where

D = inside pipe diameter
V = fluid velocity
υ = kinematic viscosity

The Reynolds number may also be calculated as

$$N_{Re} = \rho DV/\mu \quad \textbf{(EQ 10)}$$

where

ρ = fluid specific gravity
μ = dynamic viscosity

While the Reynolds number is dimensionless, care must be taken to use consistent units in its computation and use.

The transition from laminar to turbulent flow begins at a Reynolds number of about 2,000. This is called the critical Reynolds number and is important in computing fluid friction losses in pipes. In fact, critical Reynolds numbers may vary substantially with different fluids, but engineering practice assumes that laminar flow occurs below a Reynolds number of 2,000, turbulent flow occurs above 4,000, and a transition zone exists between these values. In the transition zone, both laminar and turbulent flow occur. Examination of the expressions for the Reynolds number shows that the number increases with pipe size, fluid flow velocity, and specific gravity, whereas it decreases with viscosity.

MINERAL PROCESSING APPLICATIONS

It may initially seem desirable to transport all fluids in laminar flow, since in laminar flow no energy is lost to turbulence. However, this is not the case. Almost all the fluids handled in mineral processing and hydrometallurgical processing fall into one of three categories:

1. Liquids and solutions of soluble solids, which are pumped in turbulent flow at low velocities from 0.5–2 m/s. This usually provides an optimal compromise between operating costs, which result from pipe friction losses, and capital costs for pumps and piping.
2. Slurries of liquids and settling solids, which are pumped in turbulent flow at higher velocities, from 2–5 m/s or more. Although higher flow velocity results in increased friction losses (and consequently higher operating costs), it keeps the solids in suspension, preventing pipe blockages.
3. Slurries of liquids and fine, non-settling solids usually form homogeneous slurries, which behave as Bingham plastic slurries, which are described below. These are pumped at intermediate velocities between 1 and 3 m/s, with Reynolds numbers close to 2,000, and exhibit near laminar flow.

The effects of viscosity and rheology on laminar and turbulent flow in pipes are discussed in some detail. This discussion, and the accompanying example, are closely adapted, with permission, from Grzina and Roudnev (2002).

Laminar Flow in Pipes

Recall that the viscous behavior of a Newtonian fluid is linear, as shown in Figure 2. When a shear force is applied at the boundary of a fluid, the fluid begins to move in the direction of the force, developing shear stress between adjacent layers. The dynamic viscosity μ will be the slope of the a graph showing the shear stress τ versus the velocity gradient or shear rate dv/dy, thus

$$\tau = \mu\, dv/dy \quad \textbf{(EQ 11)}$$

where y is the incremental distance in the radial direction.

When a pressure differential P is applied to a fluid over a pipe length L, in a pipe with radius R and diameter D, a frictional shear stress τ_w is generated between the fluid and the pipe at the wall of the pipe, as shown in Figure 10. The axial force causing fluid motion is given by $PD2\pi/4$, and the force resisting this motion (i.e., the wall friction) is given by $\tau_w D\pi L$. Equating these forces and rearranging terms gives the shear stress at the wall:

$$\tau_w = PD/(4L) \quad \textbf{(EQ 12)}$$

All of this happens only in laminar flow. However, in both laminar and turbulent flow, the shear stress τ decreases linearly from τ_w at the pipe wall to zero at the centerline.

The solution of the differential equation (Eq. 11) for laminar flow yields a parabolic velocity distribution with local velocities $v = 0$ at the wall and $v = v_x$ (maximum) on the centerline, as shown in Figure 10. The average velocity of flow V is equal to one-half of v_x.

Pipe friction loss is also given by the Hagen–Poiseuille equation (Eq. 4):

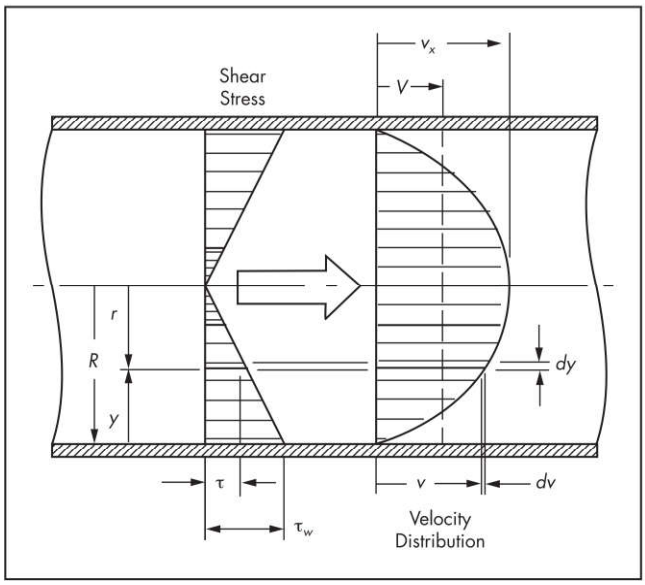
$$P = 32\mu LV/D^2 \quad \textbf{(EQ 13)}$$

Combining this with Equation 11 gives

$$\tau_w = \mu 8V/D \quad \textbf{(EQ 14)}$$

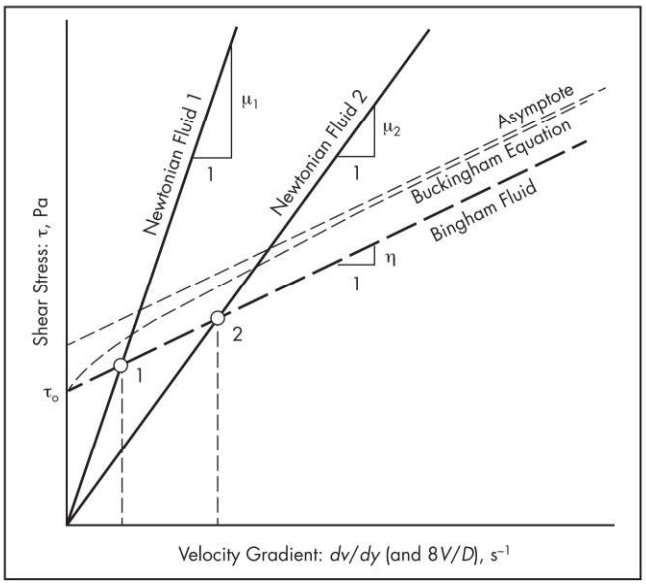
and comparing Equation 14 with Equation 11 shows that the velocity gradient is equal to

$$dv/dy = 8V/D \quad \textbf{(EQ 15)}$$



Source: Grzina et al. 2002

Figure 10 Newtonian laminar flow velocity profile in pipe



Source: Grzina et al. 2002

Figure 11 Shear stress diagram

In laboratory tests, the flow through a pipe is varied, and velocities and corresponding pressure losses are measured. A plot of these measurements, τ_w (=$PD/4L$) versus $8V/D$, will be linear for a Newtonian fluid, and the slope of that line will provide a good estimate of the dynamic viscosity, μ.

FLOW OF BINGHAM FLUIDS IN PIPES

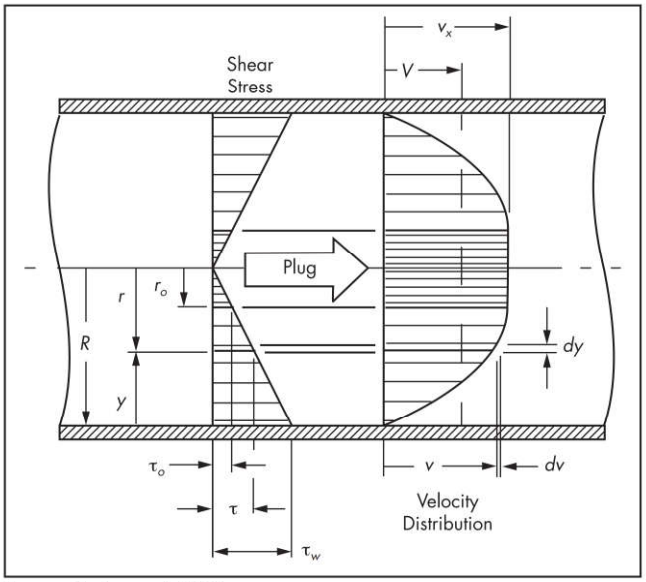
A Bingham fluid can be described as a Newtonian fluid with an additional parameter, namely a *yield stress*, τ_o. This material behaves like a jelly when stationary and like a fluid when moving. If a shear stress below τ_o is applied to it, it flexes like jelly, and when the stress is removed, it returns to its original shape. However, if the applied shear stress is above τ_o, the material begins to flow. A well-known example of a Bingham fluid is the red mud tailings produced in the Bayer process for recovering alumina from bauxite.

A plot of τ against dv/dy yields a straight line, which intercepts the τ-axis at τ_o (greater than zero), and has a slope η known as the *coefficient of rigidity*, as shown in Figure 11. The equation of this line is

$$\tau = \tau_o + \eta dv/dy \qquad \textbf{(EQ 16)}$$

Figure 11 also shows the behavior of two Newtonian fluids, 1 and 2, whose dynamic viscosities are μ_1 and μ_2, respectively. The lines intersect the Bingham fluid line at points 1 and 2, and so it can be said that the Bingham fluid has two *apparent viscosities*, μ_{a1} and μ_{a2}, identical in value to μ_1 and μ_2. Similarly, an infinite number of apparent viscosities can be located on a Bingham fluid line, but only two points are needed to define the slope η.

Shear stress τ of a flowing Bingham fluid in a pipe varies from τ_w at the wall to zero at the centerline, just like in Newtonian fluids. Solution of the differential equation (Eq. 16) yields a parabolic velocity distribution in an annulus between the pipe wall and an inner radius r_o, at which $\tau = \tau_o$, as shown in Figure 12.



Source: Grzina et al. 2002

Figure 12 Bingham laminar pipe velocity profile

The maximum local velocity v_x is at r_o and the entire central plug also flows with this velocity. The average velocity of flow is given by

$$V = D\tau_w/(8\eta)\,[1 - 4/3(\tau_o/\tau_w) + 1/3(\tau_o/\tau_w)^4] \qquad \textbf{(EQ 17)}$$

Equation 17 is known as the Buckingham equation, after its developer. The ratio τ_o/τ_w is always less than one, so $(\tau_o/\tau_w)^4$ is, of course, also smaller. Neglecting this term and rearranging the remaining terms gives

$$\tau_w = 4/3\tau_o + \eta 8V/D \qquad \textbf{(EQ 18)}$$

Setting $dv/dy = 8V/D$, Equation 18 produces the line labeled *asymptote* in Figure 11, parallel to the line described by Equation 16, but higher by an amount of $\tau_o/3$.

The Buckingham equation (Eq. 17) produces a true curve of the fluid behavior, shown as a dotted line between the asymptote and the line of Equation 18. However, this equation cannot be used to find τ_o and η because it already contains them. Values of τ_o and η must be found using Equation 16 or 18. Both yield reasonable solutions, and the simpler of the two, Equation 16, is usually selected. The value of dv/dy is set to $8V/D$ in Figure 11, which then becomes a pseudo shear stress graph. Test points usually fall on a straight line and it is then a simple matter to measure its slope η and its vertical intercept τ_o. Based on this procedure, Equation 16 for Bingham fluids is modified to

$$\tau_w = \tau_o + \eta 8V/D \quad \textbf{(EQ 19)}$$

Equation 19 and Figure 11 can be used to derive an expression for the apparent dynamic viscosity:

$$\mu_a = \eta + \tau_o/(8V/D) \quad \textbf{(EQ 20)}$$

This value can be used in the equation for Reynolds number,

$$N_{\mathrm{Re}} = \rho_m DV/\mu_a \quad \textbf{(EQ 21)}$$

where ρ_m is the specific gravity of the fluid mixture. The Reynolds number is used to calculate V_c, the critical average velocity at the end of the laminar flow range. This usually occurs at a Reynolds number of 2,000. After some substitutions and manipulation of terms, the critical velocity can be calculated, thus

$$V_c = X_1 + \sqrt{(X_1^2 + X_2)} \quad \textbf{(EQ 22)}$$

where

$$X_1 = \eta N_{\mathrm{Re}}/(2\rho_m D) \quad \textbf{(EQ 23)}$$

and

$$X_2 = \tau_o N_{\mathrm{Re}}/(8\rho_m) \quad \textbf{(EQ 24)}$$

At velocities lower than V_c, flow is laminar, and total head H_m varies relatively little with changes of flow rate Q. Pumping costs are therefore approximately directly proportional to flow rate. At velocities higher than V_c, flow is turbulent and head and pumping costs increase proportionally to the square of the flow rate. The most economical pumping velocity is therefore the one nearest to V_c.

Example: Pumping a Bingham Slurry

Slurries of water and finely ground limestone are used in the production of cement. These slurries are usually Bingham plastic fluids. To determine the lowest-cost flow rates for various pipe diameters and slurry densities, a pilot plant was built comprising a slurry pump, a holding tank, flow control and measuring equipment, instrumentation, two 100-m-long pipes of 0.150- and 0.200-m diameter, return pipes, and accessories. The static head was equal to zero.

The solids were limestone (88%) and clay (12%) with an overall density of $\rho_m = 2{,}650$ kg/m^3. The mass screen analysis is shown in Table 1 and the particle size distribution is plotted in Figure 13. The d_{50} particle size was 20 µm.

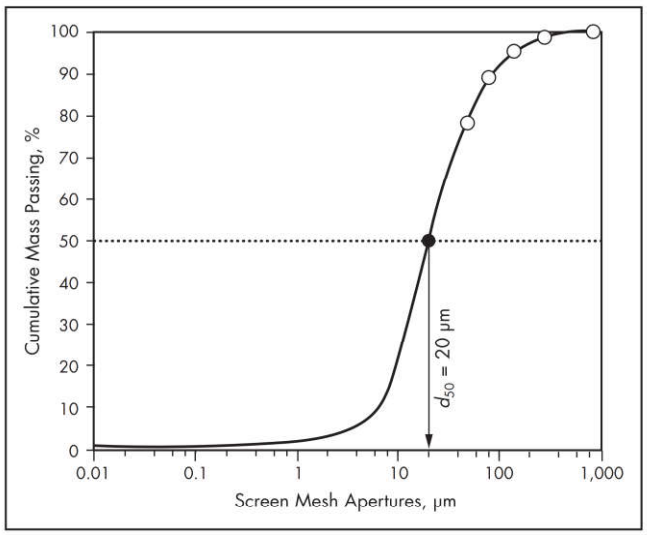
Slurries for testing were mixed and prepared on-site to C_w concentrations of 50%–65%, in 5% increments. Flow rates, pressure losses, slurry temperatures, and C_w were recorded. Hundreds of test points were collected and plotted and curves of constant C_w were fitted through all the points. In this example, only the slurry with a C_w of 65% (a density ρ_m = 1,680 kg/m^3) is described. A few data points from the tests are listed in Tables 2 and 3. These points were selected to include the transition from laminar to turbulent flow.

Figure 14 is a plot of the (Q, H_m) test points, in which the shallow-angled portions represent laminar flow and the

Table 1 Mass screen analysis

Screen Opening Size, µm	Cumulative % Passing
51	78
88	89
149	96
297	99
841	100

Adapted from Grzina et al. 2002



Source: Grzina et al. 2002

Figure 13 Size distribution of solids in slurry

Table 2 Tests with 0.150-m-diameter pipe

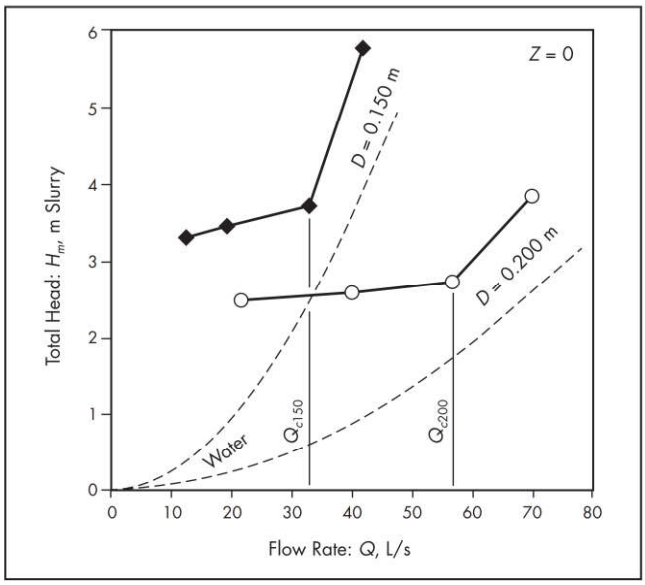
Point	V, m/s	Q, L/s	H_m, m slurry	$8V/D$, s^{-1}	τ_w, Pa
1	0.67	12	3.37	35.7	20.8
2	1.06	19	3.50	56.5	21.6
3	1.86	33	3.76	99.2	23.2
4	2.38	42	5.85	126.9	36.1

Adapted from Grzina et al. 2002

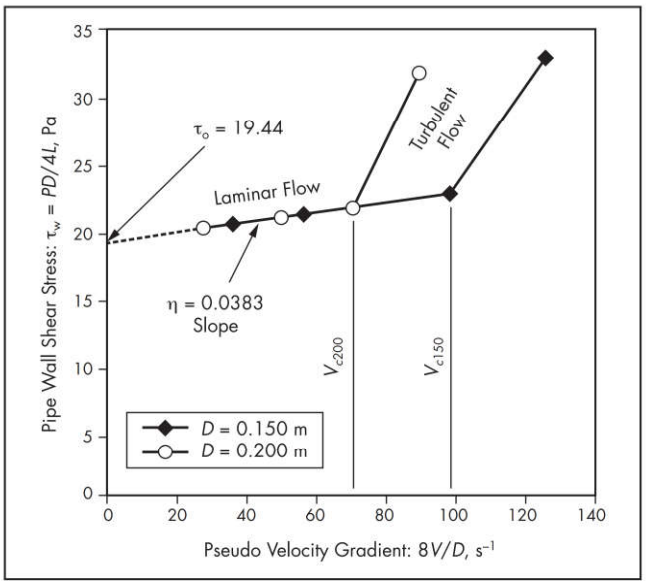
Table 3 Tests with 0.200-m-diameter pipe

Point	V, m/s	Q, L/s	H_m, m slurry	$8V/d$, s^{-1}	τ_w, Pa
1	0.68	21	2.49	27.2	20.5
2	1.28	40	2.60	51.2	21.4
3	1.82	57	2.70	72.8	22.2
4	2.23	70	3.86	89.2	31.8

Adapted from Grzina et al. 2002



Source: Grzina et al. 2002
Figure 14 Pipe friction losses



Source: Grzina et al. 2002
Figure 15 Rheology graph

steeper portions, turbulent flow. Water friction curves for the same pipes are also shown for comparison, as determined from the Moody diagram shown in Figure 2 of Chapter 5.3, "Pumps." Tables 2 and 3 also show derived values $8V/D$ and τ_w [$=PD/(4L) = g\rho_m H_m D/(4L)$]. These points are plotted in Figure 15 to show the slurry rheology in this case.

Laminar Flow

The coefficient of rigidity η may be determined from the test values by calculating the slope of the laminar-flow portion of the rheology graph shown in Figure 15. The endpoints of the laminar-flow line are point 1 of the graph for 200-m pipe (27.2, 20.2) and point 3 of the graph for the 0.150-m pipe (99.2, 23.2). The coefficient of rigidity is then given by

$$\eta = (23.2 - 20.5)/(99.2 - 27.2) = 0.0375 \text{ Pa·s} \quad \textbf{(EQ 24)}$$

The yield stress τ_o is calculated with Equation 18, using any point on the laminar flow line. For example, using point 2 of the graph for the 0.200-m pipe (51.2, 21.4):

$$\tau_o = \tau_w - \eta 8V/D = 21.40 - (0.0375)(51.2) = 19.48 \text{ Pa} \quad \textbf{(EQ 25)}$$

The critical pipeline velocities V_c for the various pipe diameters can now be determined. As mentioned, critical velocity for a slurry of this type usually occurs at a Reynolds number of about 2,000. The observed rheologies shown in Figure 15 show the critical velocity at point 3 for both pipe diameters. The Reynolds numbers calculated for these two points using Equation 21 are 2,004 and 1,961 for the 0.150-m pipe and 0.200-m pipe, respectively—reasonably close to the rule-of-thumb value 2,000.

Critical velocities may be calculated from Equations 22–24 for four pipe sizes, using a Reynolds number of 2,000. Those values are listed in Table 4, with corresponding critical volumetric flow rates Q_c.

Table 4 Critical flow rates

D, m	V_c, m/s	Q_c, L/s
0.100	1.94	15.2
0.150	1.86	32.9
0.200	1.82	57.2

Adapted from Grzina et al. 2002

REFERENCES

Bakshi, A.K., Eisele, T.C., and Kawatra, S.K. 1999. On-line rheometer for mineral slurries. SME Preprint No. 99-82. Littleton, CO: SME.

Cowper, N.T. 1992. A newly developed and commercially proven on-line viscometer offers significant benefits in mineral processing. SME Preprint No. 92-139. Littleton, CO: SME.

Drahm, W., and Matt, C. 2004. Coriolis mass flowmeter with direct in-line viscosity measurement. *S. Afr. Flow Measure. Control* (July).

Endress+Hauser. 2018. Proline Promass 80I, 83I Coriolis flowmeters. https://www.us.endress.com/en/Field-instruments-overview/Flow-measurement-product-overview/Product-Coriolis-flowmeter-Proline-Promass-83I. Accessed June 2018.

Grzina, A., and Roudnev, A. 2002. *Slurry Pumping Manual*, 1st ed. Warman International Ltd. www.pumpfundamentals.com/slurry/WeirSlurryPumpingHandbook.pdf.

Hollinderbäumer, E.W., and Mez, W. 1998. Viscosity controlled production of high-concentration backfill pastes. In *Proceedings of AusIMM '98—The Mining Cycle.* Melbourne, Victoria: Australasian Institute of Mining and Metallurgy.

Joseasorrentino. 2014. Transition from laminar flow to turbulent flow [Wikimedia]. https://commons.wikimedia.org/wiki/File:Transicion_laminar_a_turbulento.png.

Kalotay, P.Z., and Van Cleve, C.B. 1994. Viscometer for sanitary applications. U.S. Patent 5,359,881.

Kim, C.H., and Liptak, B.G. 2003a. Viscometers—Application and selection. In *Instrument Engineers' Handbook: Process Measurement and Analysis*, 4th ed. Vol. I. Edited by B.G. Liptak. Boca Raton, FL: CRC Press.

Kim, C.H., and Liptak, B.G. 2003b. Viscometers—Laboratory. In *Instrument Engineers' Handbook: Process Measurement and Analysis*, 4th ed. Vol. I. Edited by B.G. Liptak. Boca Raton, FL: CRC Press.

Kim, C.H., Liptak, B.G, and Jamison, J.E. 2003. Viscometers—Industrial. In *Instrument Engineers' Handbook: Process Measurement and Analysis*, 4th ed. Vol. I. Edited by B.G. Liptak. Boca Raton, FL: CRC Press.

Napier-Munn, T.J., Reeves, T.J., and Hansen, J.O. 1989. The monitoring of medium rheology in dense medium cyclone plants. *Proc. Aus. Inst. Min. Metall.* 294(3):85–93.

Reeves, T. J. 1985. On-line viscometer for mineral slurries. *Trans. Inst. Min. Metall.* 94:C201–C208.

Tilton, J.N. 2007. Fluid and particle dynamics. In *Perry's Chemical Engineers' Handbook*, 8th ed. Edited by D.W. Green and R.H. Perry. New York: McGraw-Hill.

Van Cleve, C.B., and Loving, R.A. 1997. Coriolis viscometer using parallel connected Coriolis mass flowmeters. U.S. Patent 5,661,232.

Van Wazer, J.R., Lyons, J.W., Kim, K.Y., and Colwell, R.E. 1963. *Viscosity and Flow Measurement*. New York: Interscience.