CHAPTER 2.4

Metal Accounting

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Metal accounting is still treated as a "poor cousin" at many sites even though the situation has improved in recent years. However, a dependable metal accounting system should provide a reliable basis for estimating production, losses, and important key performance indicators (KPIs). Metal accounting also provides a key tool in the search for "low-hanging fruit" and is probably the only way to confirm with some reasonable degree of confidence that they might have been harvested.

A five-year, industry-funded, collaborative project (AMIRA International Project 754 [P754], Metal Accounting and Reconciliation) was undertaken from 2004 to 2009 to address the challenges of metal accounting. The project produced a draft code of practice (AMIRA International 2007), trained seven graduate students, and culminated with publication of a textbook (Morrison 2008). The project carried out detailed metal accounting studies at sponsor-nominated sites.

These studies disproved some common myths and identified some serious issues. Metal accounting problems are often attributed to poor sampling practice. In a few cases, sampling was an issue. However, the industry is well supported by sampling consultants and International Organization for Standardization (ISO)-certified laboratories. Hence, at many sites, sampling and assaying are well executed and no longer offer unresolved issues. Nevertheless, it is still worth comparing the sampling methods in use for metal accounting at a particular site with those recommended in Chapter 1.8, "Sampling Practice and Considerations," and rectifying any shortcomings.

Just how much material has been processed and produced is, of course, critical to metal accounting. The belt weightometer is usually regarded as the "gold" standard. Modern belt weighers are sophisticated, robust, and generally excellent measurement devices when correctly installed and operated. In practice, they are rarely installed properly, which seriously compromises their potential performance. Standards of operation also vary widely. However, the biggest challenge for metal accounting is that it is intrinsically a collision between technical and financial cultures.

AMIRA P754 developed a systematic approach to aligning these cultures so that the estimated balances would be statistically identical. Hence, only one set of numbers need be considered for reporting, for generation of KPIs, and, more importantly, for decision support.

In short, the process of metal accounting should transform measured data into performance information that can be used for reliable decision making. This chapter works through the key features of the process and provides some practical examples.

THE BASICS

The underlying purpose of metal accounting is to "process" a set of measurements and historical data into a set of self-consistent numbers that can be used as a sound basis for making decisions. Hence, a more succinct description of *metal accounting* is to "turn data into information."

It is also necessary to select the "best" streams for sampling and mass measurement. The obvious streams are the feed, concentrates, and tailings for the total plant. However, measuring each stage of a complex plant may provide useful information about its performance and aid in diagnosis of problems. Mass measurements and sampling points are also needed wherever a transfer of custody occurs. Masses and metal contents measured in more than one way may seem excessive but are vital to the development of cumulative sums, or "cusums," which allow biases to be detected and tracked over time.

Metal accounting information is typically used as the basis for a series of period-based reports. These reports usually provide estimates of input to the process and where those inputs report to, as well as changes in accumulation (i.e., stockpiles, bins, or tanks) between the beginning and the end of the reporting period. These estimates should be the "best available" in terms of the reliability of the measured data within the period and information related to previous operating periods.

As there will usually be many more measurements than are required to generate a single balance, it is tempting to create different balances for different purposes. This means

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that many "best estimates" can be generated. They need not be self-consistent, and decision making will be influenced by opinion about which balances might be best.

Metal accounting also covers the transfer process between different parts of an organization (mine to concentrator) and to other organizations (concentrator to smelter.) The relationships with external organizations are usually defined by comprehensive sales contracts. Within organizations, these contracts are often informal. However, the potential for internal strife can usually be reduced with at least a semiformal "contract" describing the points at which custody is transferred and how measurements and calculations are to be carried out.

As with all contractual processes, especially those that involve commercial transfers, some definitions acceptable to both parties are essential. Hence, the major challenges for metal accounting are measurement (and quality of measurement) as well as management of custody transfers between various parties.

Given that technical and financial cultures are different, metal accounting also involves managing those relationships in a structured and cooperative manner, as they offer substantial opportunities for confusion and conflict.

Until recently, there have not been any broadly applicable definitions or a robust, published methodology available. An important driver for development of the code was to improve corporate governance. Developing a common language was also important. Hence, the code developed an extensive glossary with detailed definitions of key terms and concepts.

THE DRAFT CODE OF PRACTICE

For many years, there has been some degree of acceptance that as much as 15% of the metal in a body of ore might fail to appear at the concentrator. In an era where measurements were few (at both mine and concentrator) and mechanized samplers even fewer, this might have been acceptable. In the current era, hugely better mass measurement and fully automated samplers are available, as well as better assaying and essentially unlimited computational capabilities. Hence, there are no valid excuses for using 19th-century approaches in the 21st century. In a few cases, application of the code might have detected fraud more quickly, but in almost all cases it will likely identify problems and opportunities as they develop. Furthermore, a sound metal accounting system linked to records of which ore types were being processed in each period offers a basis for a sound production plan.

The development of the draft code is well documented (Morrison and Gaylard 2008). Hence, only a brief summary of the objectives and principles is covered in this chapter. The code itself is freely available from AMIRA International (AMIRA 2007).

The draft code's glossary also includes definitions that are used in the remainder of this chapter. Formal definitions drawn from the code are included in this chapter.

The code defines *metal accounting* and *reconciliation* as follows (AMIRA 2007). These definitions and all quotations from the code are provided by kind permission of AMIRA International:

> **Metal Accounting.** The system whereby selected process data (pertaining to metals of economic interest) is collected from various sources including mass measurement and analysis and transformed into a coherent report format that is delivered in a timely fashion in order to meet specified reporting requirements.
>
> **Reconciliation.** A metallurgical balance which relates production of saleable and reject or waste materials from a process back to its source as ore or other feed material. It should be provided with defined and stated errors as for any other metallurgical balance.

The code is based on the following 10 principles (AMIRA 2007):

1. The metal accounting system must be based on accurate measurements of mass and metal content. It must be based on a full Check in-Check out system using the Best Practices as defined in the Code, to produce an on-going metal/commodity balance for the operation. The system must be integrated with management information systems, providing a one-way transfer of information to these systems as required.
2. The system must be consistent and transparent and the source of all input data to the system must be clear and understood by all users of the system. The design and specification of the system must incorporate the outcomes of a risk assessment of all aspects of the metal accounting process.
3. The accounting procedures must be well documented and user friendly for easy application by plant personnel, to avoid the system becoming dependent on one person, and must incorporate clear controls and audit trails. Calculation procedures must be in line with the requirements set out in the Code and consistent at all times with clear rules for handling the data.
4. The system must be subject to regular internal and external audits and reviews as specified in the relevant sections of the Code to ensure compliance with all aspects of the defined procedures. These reviews must include assessments of the associated risks and recommendations for their mitigation, when the agreed risk is exceeded.
5. Accounting results must be made available timeously, to meet operational reporting needs, including the provision of information for other management information systems, and to facilitate corrective action or investigation. A detailed report must be issued on each investigation, together with management's response to rectify the problem. When completed, the plan and resulting action must be signed-off by the Competent Person.
6. Where provisional data has to be used to meet reporting deadlines, such as at month ends when analytical turn-around times could prevent the prompt issuing of the monthly report, clear procedures and levels of authorisation for the subsequent replacement of the provisional data with actual data must be defined. Where rogue

data is detected, such as incorrect data transfer or identified malfunction of equipment, the procedures to be followed, together with the levels of authorisation must be in place.

7. The system must generate sufficient data to allow for data verification, the handling of metal/commodity transfers, the reconciliation of metal/commodity balances, and the measurement of accuracies and error detection, which should not show any consistent bias. Measurement and computational procedures must be free of a defined critical level of bias.
8. Target accuracies for the mass measurements and the sampling and analyses must be identified for each input and output stream used for accounting purposes. The actual accuracies for metal recoveries, based on the actual accuracies, as determined by statistical analysis, of the raw data, achieved over a company's reporting period must be stated in the report to the Company's Audit Committee. Should these show a bias that the Company considers material to its results, the fact must be reported to shareholders.
9. In-process inventory figures must be verified by physical stock-takes at prescribed intervals, at least annually, and procedures and authority levels for stock adjustments and the treatment of unaccounted losses or gains must be clearly defined.
10. The metal accounting system must ensure that every effort is made to identify any bias that may occur, as rapidly as possible, and eliminate or reduce to an acceptable level the source of bias from all measurement, sampling and analytical procedures, when the source is identified.

As the project to develop the draft code neared its conclusion, it became clear that the code would provide a strong guide to what a metal accounting system should achieve but not a great deal of guidance about how to actually achieve the desired outcomes. Therefore, the project team and some invited experts produced a textbook (Morrison 2008) that provided practical guidance on how to account for a wide range of processes.

The underlying process of metal accounting is custody transfer. The remainder of this chapter considers custody transfer, followed by measurement (mass and sampling), and approaches to calculation. How to report the outcomes of the system and to detect longer-term errors are considered.

CUSTODY TRANSFER (CHECK IN–CHECK OUT)

A custody transfer is an ancient process where a seller and a buyer agree on what is to be bought, how it is to be measured, and what the price will be based on. The formal definition from the code is as follows (AMIRA 2007):

> **Check In-Check Out System.** The system whereby all streams into and out of the Process or Plant, for which the balance is being performed, are measured, sampled and analysed.

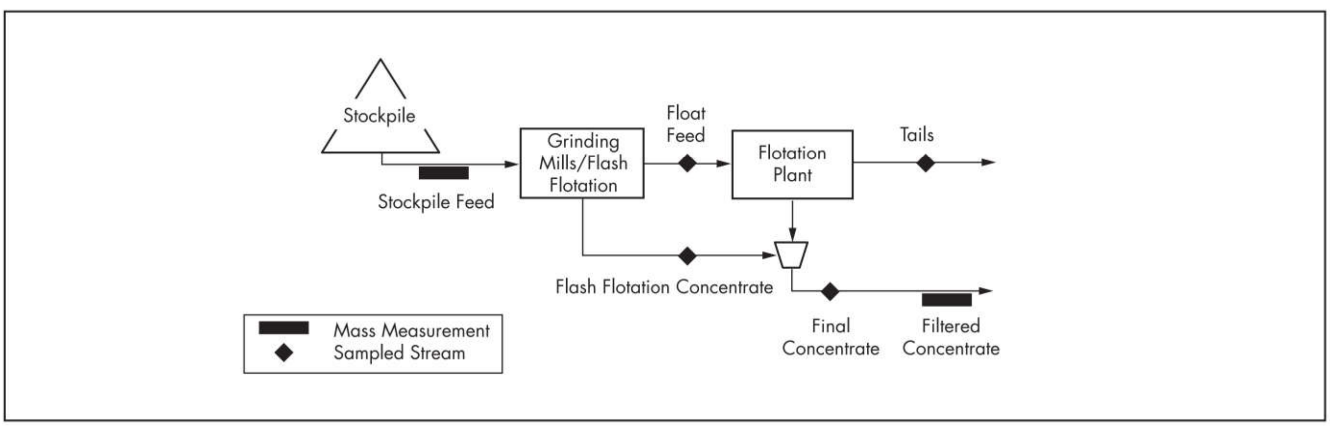
In essence, a formal custody transfer is very much like a sales contract for a concentrate or a contract for toll treatment of a parcel of ore. Applying an agreed set of definitions and a check in–check out (CI/CO) process to the transfers will greatly reduce variations due to "opinions" of both buyer and seller. As noted earlier, it is certainly worth considering at least an informal "contract" within organizations and it should still be written down.

Occasionally, the transfer process will break down. In these cases, an exception report is required (AMIRA 2007):

> **Exception Report.** A report generated, in terms of this Code, whereby each non-compliance with the requirements of this Code is motivated and approved.

This is similar to a yellow or red tag for a process plant malfunction and is likewise a problem to be diagnosed and fixed as quickly as possible.

In the flow sheet shown in Figure 1, the CI/CO points are fairly clear. The transfer from mine to concentrator happens at the mass measurement (and sampling) point between the stockpile and the mill. Within the concentrator, the CI/CO points between grinding and conventional flotation, and conventional flotation and tailings management, are also obvious. However, flash flotation is a part of the milling circuit with a major impact on overall flotation performance. It will



Source: Jansen et al. 2007, reprinted with permission from the Australasian Institute of Mining and Metallurgy

Figure 1 Overview flow sheet and measurement points at the Northparkes copper–gold concentrator

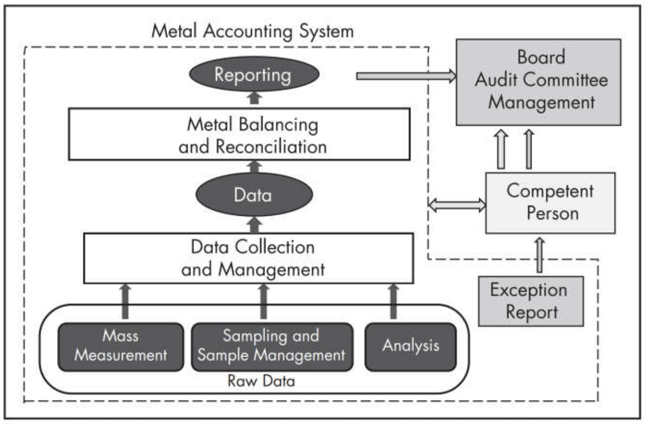


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Figure 2 Structure and data flows within a typical metal accounting system

need some careful consideration for effective custody transfer boundary. One possible approach is provided as an example.

MEASUREMENT

The data-gathering component of a metal accounting system is mostly concerned with measurement. An accounting system must have at least one (and preferably several) point for mass measurement. A sampling and assaying procedure will usually be associated with each measurement point and always with each transfer of custody, as shown in Figure 2.

Measurement Quality

All measurement and sampling processes are associated with an error distribution. (See Chapter 1.8, "Sampling Practice and Considerations," for further guidance about sampling.) Given that metal accounting is very much concerned with how well measurements are defined, it is worth briefly considering the types of errors that contribute to the quality of a measurement and to the credibility of calculated outcomes based on those measurements.

Three types of error are of particular interest: accuracy, precision, and bias. The code definition for *accuracy* is rather long-winded, but it is also self-explanatory (AMIRA 2007):

> **Accuracy.** A measurement is accurate if it, or the average of a number of measurements, is close to the true value. In metallurgical operations this true value is unknown.
>
> In addition, there is often misunderstanding between the terms accuracy and precision, which is the measure of the spread of a number of measurements around their mean value.
>
> For these reasons, it has been decided to adopt the following definition for accuracy, which incorporates the concept of precision, and is based on that given in ISO 5725-1:1994.
>
> A measurement that is accurate is one that is free of bias and has a dispersion (standard deviation) that is lower than a defined dispersion or indeed a probability density of a particular nature. The level of dispersion or the nature of the probability density is defined with the purpose of separating measurements that are entirely fit for a particular purpose or use and those that are not.

The definition of *precision* is also self-explanatory. Many engineers use *reproducibility* interchangeably with *precision*. Because metal accounting is usually concerned with absolutes in terms of payment for product in a custody transfer, one must be most concerned about accuracy in the short term and avoidance of bias in the longer term.

Precision of a measurement depends on the closeness of the outcomes of a repeated measurement or test procedure. Hence, it depends only on the distribution of random errors and not on any relationship to a "true" value. It is usually expressed as the standard deviation of the test results. That is, by a measure of imprecision. The code defines *precision* as follows (AMIRA 2007):

> **Precision.** Precise measurements have a dispersion about their mean value, which is lower than a defined dispersion or indeed a probability density of a particular nature. The level of dispersion or the nature of the probability density is defined with the purpose of separating measurements that are entirely fit for a particular purpose or use and those that are not. A precise measurement may not be accurate; its mean may differ from the true value of the measured quantity by an arbitrary amount. Standards often quantify the term precision as a value corresponding to the magnitude of a 95% confidence interval around a result.

If the n measurements of the quantity to be estimated are normally distributed, "the interval is $\pm ts$, where t is the two-sided Student-t value at 95% confidence and $n - 1$ degrees of freedom, and s is the estimate of the standard deviation" of the estimated quantity.

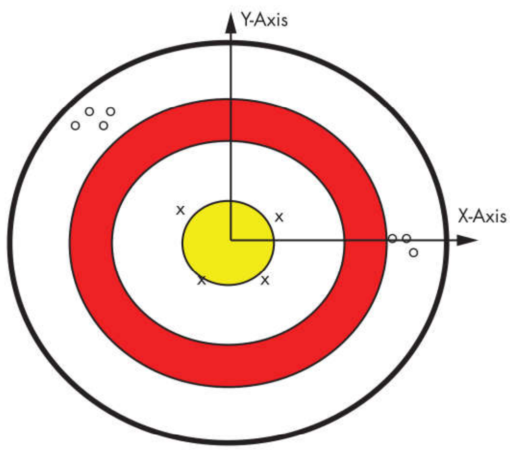
Metal accounting is also concerned with long-term quality of data. Hence, bias—or the detection and correction of it—is also critical. The formal definition is perhaps not as clear as the others but may become clear after the reading the discussion in the "Detecting Bias" section later in this chapter. The code definition for *bias* follows (AMIRA 2007):

> **Bias.** Bias is the difference between the mean result of one or more measurements and the true value of the quantity being measured. It also can be seen as an ongoing difference between two, or more, measuring systems. (e.g. sender's and receiver's weights, the analysis of the same sample by two laboratories etc.)

Figure 3 summarizes these error types.

Mass Measurement

Mass measurements can be continuous (flow rates on conveyors or in pipes), discrete (by mine ore truck or concentrate rail wagon), or more or less static (stockpile or tank surveys). For continuous measurements, the integral of the measurement over an accounting period is the key piece of data. However, while the integral of dry mass is the most important, most systems measure wet mass.



Note: The x's are accurate but are not very precise. The small circles are less accurate and more precise but exhibit bias in X alone in the right-hand set and in both X and Y in the other set.

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Figure 3 Accuracy, precision, and bias

Belt weightometers (or scales) are usually considered to be the "gold" standard. These devices are certainly capable of high-quality measurements if correctly installed, calibrated, and maintained. One of the more startling outcomes of the investigations carried out during the development of the code was that these requirements are very rarely fully satisfied.

Hence, a good first step toward a robust metal accounting system is to check the manufacturer's guide for recommended installation requirements for the weigh scale. The code contains five pages of these requirements. Dominant areas of noncompliance include using a weighing point that is not on a horizontal section of the belt and failing to keep the weigh frame clean. The latter is important because the weigher will indicate that more ore has been treated than has actually passed over the frame. A quick check at the start of each shift for rocks that have "fallen" onto the weigh frame is good practice.

In an ideal situation, a belt conveyor should be able to be diverted to a truck that can then be weighed on a weigh bridge that has been "certified for trade" by the relevant authority. This is not possible for many primary ore feed conveyors. A more workable alternative is to weigh several truckloads and then load them onto the belt. A roller chain for calibration should be standard, as should regular, manual belt speed checks—given the wide range of ways in which belt speed sensors can malfunction. The roller chain may be described as "certified," which means it has a well-defined mass per unit length of chain. This does not mean that the measurement the weigher produces is "certified for commercial use," as considered further below.

Static weights can be attached to the weigh frame to check it for zero and ranging errors. This will check if the load cell is drifting, but it will not help with variable belt tension or belt speed errors.

Belt scale manufacturers specify measurement quality as an "error" that is a percentage of full scale. They are often reluctant to state just how many standard deviations the claimed error represents. Assuming that the stated error represents one standard deviation is a sensibly conservative point to start at, given the myriad possible sources of error.

Consequently, about two-thirds of the time, the belt integral should be expected for, say, a shift to be within plus or minus the stated error range. Obviously, a well-loaded belt will be better measured (i.e., be subject to a lower relative standard deviation, as the specified error is absolute) than a lightly loaded one. It should be made clear that the belt loading should be as steady as possible with only slow variation. This is achievable with multiple feeders drawing from a stockpile that is not close to empty. With direct (and always intermittent) loading from a truck or front-end loader, the belt load per meter will vary widely and reduce the reliability of the mass measurement. Almost all belt weighers produce an increment. This means that the weigher integrator will advance by one unit. An increment results from the product of belt movement and weigh-frame load. Some also produce an analog signal for an instantaneous product of weigh frame load and belt speed. Integrating that signal in the control system—smoothed or not—is not recommended because the weigher integrator itself should be the most reliable. If the weigh frame analog signal is available, rapid fluctuation will usually indicate problems with belt loading or the hardware. If the signal is reaching its maximum, the belt load is certainly high but not actually measured.

So-called automated calibration, where the belt is empty or loaded with a roller chain and is run for several revolutions, should provide an indication of just how much the zero point and slope have been adjusted. Otherwise, it becomes very difficult to track variations and biases in the integrated mass records.

International standards do provide further guidance, but ore-feed weigh scales are rarely certified for commercial use. *Certification* is also defined in the code (AMIRA 2007):

> **Certification (Test).** A calibration of a scale or flow meter in terms of accepted standards defined by the relevant government authority to permit its use for trade purposes and custody transfer.

A comprehensive review of weightometer application—and mass measurement in general—is provided by Wortley (2009). Wortley also contributed to the code and to the textbook. By far, the most comprehensive reference for mass measurement is by Colijn (1983), but it is out of print and difficult to obtain.

Moisture Measurement

As the estimated metal content of a stream is based on a dry assay, measuring or estimating the moisture content of the ore feed stream is important.

Before semiautogenous milling became dominant for primary grinding, plants typically employed three stages of crushing that preceded milling; this was fairly simple because a representative sample of the final crushed product was usually a practical size for lab processing. With the almost universal dominance of semiautogenous milling, the problem has become much more challenging. An accurate measurement would require periodic large samples, which would then be crushed and subsampled. Unless the mineral of interest is of low grade but very high value, this strategy will be too expensive to install and maintain. In practice, most of the moisture is usually contained in the –6 mm size fraction. The percentage of fines can be evaluated as a proxy for moisture when multiplied by a reasonably constant factor, as almost all of the moisture is contained in the fines fraction of the stream.

Because of the challenge, a common approach (which is almost standard practice) is to combine the metal masses in tails and concentrates to estimate the feed grade to the semiautogenous grinding (SAG) mill. Hence, the downstream mass measurements become critical. This challenge is discussed further in the case study of Northparkes, reported in detail by Jansen et al. (2007) and summarized later in this chapter.

Possibilities exist for automation of moisture measurement. Measures based on capacitance and response to microwaves are commercially available. However, these meters also need to be kept calibrated, and some common minerals—magnetite and many sulfides—interfere with the measurements to some degree.

Flow Measurement

The flows of most interest in mineral processing are usually slurries. Therefore, using flowmeters that have no internal protrusions is good practice. For good-quality solids flow measurement, a combination of a magnetic flowmeter and a nuclear density gauge constitutes the traditional approach. Various acoustics-based approaches are also available, and their technologies are both improving and offering an ever-wider choice.

For almost all types of flowmeters, several pipe diameters of straight pipe should be provided for approach and retreat. Instrument manufacturers typically recommend 10 or more diameters. This is often impractical. However, it is critical that no mass holdup or any degree of partial filling is occurring within the meter. Hence, upward vertical flow is preferred—never on a gentle downward slope. For downward slopes, a weir with level measurement should be used.

If a slurry meter is installed vertically, it is also essential that there are no coarse particles in the flow that can settle at a rate approaching the flow velocity and generate misleading density measurements and consequent overestimates of flow rate. In this situation, acoustic flowmeters that depend on the Doppler effect will be biased low as they measure an average particle velocity. Hence, for accurate, integrated flow measurement, flow rate and density should be kept within specified ranges.

The ideal way to calibrate a flowmeter is by timed flow into a large tank. For slurries, the stream can be sampled to measure density. In practice, a portable ultrasonic flowmeter can often provide an adequate check.

Nuclear density gauges are calibrated assuming an average particle density. If the mineral mix varies much, the solid flow estimates can be biased in either direction. A nuclear gauge installation should allow for calibration against a pipe full of water and have a set of high-density plates that can be introduced into the beam path for a multipoint calibration.

As the pipe diameter increases with wear, flow rate and density measures will read low. This is important because the integrated error will become a bias, and flow at a particular velocity depends on diameter squared. Therefore, flow and density must be periodically calibrated for pipe wear.

For a more detailed discussion, see Section 3.2.2 of the draft code (AMIRA 2007).

Shipping Measurements

For bulk transport, there are very good train wagon weighers that can even be certified while the wagons are moving slowly through a tipping or dumping station. Truck weighers can also provide good-quality results but can only be certified for static operation.

The major transport method for many concentrates and bulk commodities is by ship. Strangely, the method of estimating net cargo mass is by surveying how far the ship sinks into the water while unloaded and loaded. Corrections are used for water temperature and salinity, but the method is inherently limited, with a typical standard deviation of about 5% relative. Its great advantage is that it is difficult to interfere with the measurement (see Section 3.4.6 of the code for more detail [AMIRA 2007].)

For bulk commodities, the purchaser often divides the measurement and payment process into two steps. A proportion of the payment (a provisional payment) is made based on the measured load and sampled assays going into a ship or onto a train. A final payment is then made on the as-received mass loadout and assays. This is a complex CI/CO process, which will be covered by a detailed sales contract. The contract will also detail how disputes are to be resolved—for example, by using another laboratory to assay duplicate samples.

Some additional factors need to be considered when sampling containers. After the material of interest is placed into the container, it becomes very difficult to take a representative sample. A continuous feed stream to a container by pipe or conveyor provides a nearly ideal situation for crosscut sampling and precise weighing. Section 4 of the code (AMIRA 2007) and Chapter 1.8, "Sampling Practice and Considerations," provide more detail on this topic. The most difficult containers to sample are stockpiles and bins. Measuring mass and composition in and out of the stockpile is the favored strategy. Frequent zero points should also be part of the system. If there are two concentrate storage tanks, they should operate alternately, not in parallel.

A large stockpile can often be managed as two smaller ones, more than doubling the number of zero points. Frequent zero points help to detect biases that may be creeping into the system if any of the calibrations suffer from drift.

ASSAYING

The assays of most interest for metal accounting are usually for water (i.e., % solids or % moisture) and for paid metals and (penalty) contaminants, as these will determine the price paid for the concentrate. For moisture, particularly in arid climates, not keeping samples sealed from dry air is often a source of bias.

As for all accounting samples, they should be cut for equal increments of mass flowing past the sampling point. The next preferred strategy is random sampling with a well-controlled average cutting frequency. Given that the processing of large samples often leads to errors in sample splitting, multistage samplers should be considered for large flows of slurry. Wills and Napier-Munn (2006) provide a good guide to the design of multistage systems.

Assaying was once an issue, as many errors can be generated in the process. Today, many laboratories are ISO 9000 certified. These labs must have strong procedures and quality assurance (QA) procedures in place. Many will also be using error models to monitor quality.

The standard practice is to process duplicate samples and do a third assay if a trigger discrepancy between the first two occurs. Section 4 of the code elaborates on QA methods (AMIRA 2007).

Error Models

Some examples of error models are given in the case study of Northparkes, reported in detail by Jansen et al. (2007) and summarized later in this chapter.

MASS BALANCING AND RECONCILIATION*

It needs to be emphasized that mass balancing for metal accounting is not the same as mass balancing of, for example, a flotation circuit survey, even though similar mathematical tools may be used. Blindly applying a standard mass balancing package is an effective way of concealing bias. Detecting and avoiding bias should always be a major objective of a balancing and reconciliation system for metal accounting.

Objectives

As previously mentioned, the code recommends a full CI/CO methodology at each transfer of custody. Therefore, any mass balancing technique must be compatible with CI/CO procedures. Perhaps the most valuable aspect of a sound mass balance is that it can be used as a go/no-go test for each CI/CO transfer. This satisfies the first principle in the 10 principles of the draft code of practice (AMIRA 2007). Another way to phrase this principle is as a question: Is there a significant discrepancy, or are the variations within expected measurement error at some agreed level of confidence?

The second objective is to pinpoint any measurement problems or biases without delay. These problems should be rectified as quickly as possible, not allegedly "corrected" by the balancing process.

To set up the mass balancing problem to suit metal accounting and reconciliation, a sound knowledge of error distributions in data measurement is required. All measurement processes—such as instrument readings, sampling and assay measurements, or any other kind—are subject to statistical error. In addition, two useful concepts need to be revisited:

1. A measurement is *accurate* if it is sufficiently close to an accepted standard. A good example is the process of certifying a weighbridge or scale for commercial use, which is a classic example of a custody transfer. A certified mass measurement device usually provides a key input to a CI/CO transfer of value.
2. A measurement is *precise* if it gives very similar results when repeated. If a precise measurement is not close to the result that is accepted as accurate, then there is bias. A small bias is not a problem in a short-term experiment, such as a flotation survey. However, where key results are accumulated over many measurement periods, a small bias will accumulate and cause problems with reconciliation and achieving fair CI/CO transfers.

Figure 3 summarizes these concepts.

One way to state the problem mathematically is to define an adjustment between each measured and balanced value. For n measurements, each of which has a counter value of i, an adjustment delta can be defined as follows:

$$\Delta_i = x_i - \bar{x}_i \qquad \textbf{(EQ 1)}$$

where x_i is each measured value, and $\bar{x}_i$ is an adjusted value of x_i.

Each adjustment can be scaled by its measurement accuracy, estimated by its standard deviation σ_i or some other measure of dispersion. The standard deviation in this case should include all of the measurement errors—including sampling, and sample preparation and analysis, where appropriate.

Now a mathematical criterion can be defined—the weighted sum of squares, WSSQ—which can be minimized to estimate a "best" set of data adjustments:

$$\text{WSSQ} = \sum_i [\Delta_i/\sigma_i]^2 \text{ subject to } C[x_i] = 0 \qquad \textbf{(EQ 2)}$$

where C is a matrix of constraints that must be satisfied to be consistent with the complete flow sheet. The flow sheet in this case includes variations in holdup in bins, stockpiles, or in the process circuit itself.

If the required adjustment delta i is small compared with the measured variable, then intuitively the required adjustment is not a *discrepancy* in CI/CO terms. For normally distributed measurement variation, the required adjustment is expected to be within plus or minus one sigma for about two-thirds of the measurement and within plus or minus two sigma for about 95% of it. Hence, adjustments of more than twice the standard deviation need to be carefully examined.

Bias is easy to include in this model but not so easy to detect:

$$\Delta_i = (x'_i + b_i) - \bar{x}_i \qquad \textbf{(EQ 3)}$$

where b_i is a bias associated with measurement i, and x'_i is the unbiased measurement. If b_i is small compared with the measurement, it is not detectable in a single data set. However, as each mass and metal content increment is accumulated, the expected relative error of each accumulated sum reduces by the inverse of the square root of the number of increments.

Consequently, the bias becomes easier to detect and, in many cases, impossible to ignore. A maximum of one bias at a time can be tested for as part of the least-squares approach. Practical approaches to bias detection are considered later in this chapter.

Solving the Problem

Equations 1 and 2 constitute a standard problem for constrained minimization of the WSSQ, provided reasonably accurate initial estimates are available. Mass balancing provides a reasonably linear problem, and many simplifications are possible. The two-product formula is the most venerable of these, but it provides no information about self-consistency. Standard techniques for error propagation can be used (Cutler and Eksteen 2006). Where multiple components are measured, the standard deviation of the mass split can be estimated.

Two methods are available to attack the general mass balancing problem. The first is to estimate mass splits based on assay differences. Appendix E of the code of practice provides an example (AMIRA 2007). This is the traditional mass balancing approach. Although it can be extended to include process holdups (Morrison and Richardson 1991), it is not generally useful beyond the processing plant.

Most custody transfers occur into or out of separation plants rather than within them. The traditional approach was

* This section is drawn from Gaylard et al. 2009, with permission from the Southern African Institute of Mining and Metallurgy.

very attractive for hand calculation because it is very computationally efficient.

With computational power available from modern computers, the second approach using simulation has much to recommend it. The simulation approach is well suited to stream splitters that provide undefined mass splits because they should not generate assay differences. For the simulation approach, each stream is considered in terms of volumetric flows, as well as of each metal and gangue of interest. Each process block (or node) is modeled as a set of splitters.

If each splitter is modeled in flow-sheet order (with iterations as required for recycle streams), the constraints of Equation 2 become implicit (i.e., they are automatically included). Hence, only the split factors and input flows need to be adjusted. Constraining split factors between 0 and 1 is also recommended. For a detailed description of both techniques, see Chapters 6 and 7 of Morrison (2008).

Reconciliation

The simulation approach is also well suited to reconciliation. The objective of reconciliation of metal accounts is to develop a set of data that is numerically consistent with the CI/CO values at its boundaries (for which the operation paid and/or was paid for) and the measured values within that scope. This can be carried out in two steps:

1. Carry out the balance and check that all values are within the expected ranges of error.
2. If step 1 is satisfied, "fix" the CI/CO values and rerun the balance.

If the measured values are still within the expected ranges, now there is a fully consistent and statistically validated set of data suitable for generation of KPIs and reports. A few values beyond ±2 standard deviations are expected.

Detecting Bias

The traditional approach is to use cumulative sums (or "cusums") of the difference between cumulative metal flows (at any point in the process chain) where they can be estimated by two more-or-less independent methods. Over time, the differences should fluctuate to about zero due to random error. A bias will cause a positive (or negative) accumulation and should be easy to detect.

For the mass balancing approach, it should be clear that any bias will become part of the data adjustments. If there is no bias, the weighted adjustment is expected to have an equal chance of being positive or negative. Even though the bias will be distributed to some degree, the adjustments will tend to be predominantly positive (or negative) if a bias develops. Hence, both cusums and weighted adjustments should be trended for early detection of bias.

APPLYING THE CODE

Processing plants come in many different configurations. However, for the purposes of metal accounting, they can be divided into several types (Table 1). Each type is suited to a particular measurement and accounting strategy. The textbook devotes a chapter to each of these applications (Morrison 2008). For this chapter, a more general case is considered, and then two typical cases are summarized.

Figure 4 shows a generic processing plant flow sheet. It receives feed from four sources (F) via four stockpiles (S). Within the plant there are several holdups (H_1–H_3). The process plant produces rejects (R) for tailings and several products that are stored in inventories (I_1–I_3). During an accounting period, total product sales (P_1–P_3) are drawn from each inventory. For this generic case, there are three valuable components with assays (c1, c2, c3) and two contaminants (c4 and c5).

Check In–Check Out Points

The trucks that deliver feed pass over a certified weightometer. That measurement provides a wet weight F to check in to each stockpile. The wet weight is converted to an estimated dry weight using an average moisture content or a truck factor based on experience. Stocks are reclaimed over a belt weightometer to provide a check-out value for feed stocks and a check-in value for the process plant.

Conveyor reclaim also provides a convenient place for sampling. The samples provide assays c1 to c5 for defined increments of process plant feed. The process block is the most interesting because it produces three products and has three holdups and a single reject product stream.

For each accounting period, the feed-stocks drawdown should be equal to the sum of the rejects and the check-out flows over a conveyor weightometer, with quality control sampling plus the net change in process holdups (H_1–H_3). H_1–H_3 might be positive or negative, while all of the mass and component increments should, of course, be positive.

The products are drawn from the inventories (I_1–I_3) on an as-required basis using a front-end loader with real-time load weighing. The trucks loaded with product exit the site over the same certified weightometer. They will typically be weighed into a customer's plant and then will be sampled as they are unloaded and assayed for quality control.

Hence, there are many CI/CO stages to be considered—for example, feed source out and into feed stocks; feed stocks out into plant and out into products and reject.

Given that all flows in and out per accounting period are measured and assayed (at least at some stage), this balance is completely arithmetical. The total masses and components masses in and out of each stage are simply added, with adjustments for changes in stock and holdups.

How well these arithmetic balances match at each stage is sometimes called "accountability." Statistical variation suggests that it will be unreasonable to expect them to match perfectly every time.

When a stockpile is empty, there is an opportunity to check the accumulated balance and make an adjustment to stock if necessary.

In some industries—typically those with high-value products—an "empty zone" is moved through the process by stopping all feed streams and measuring stocks and holdups as they empty. This is called a bubble method or audit. It will incur some production costs, but it will identify biases in terms of holdups and stocks that contain too much or too little of valuable components.

The separation plant of this generic accounting process can also be combined with the two-product approach, particularly when each product is batched to inventory. This strategy is considered in the two practical examples (case studies) that are presented later in this chapter.

ESTIMATING ACCURACY

A very useful rule called the propagation of variance (or error) allows the accuracy of calculated values to be estimated and the credibility of measurements to be assessed. The rule states

Table 1 Suggested metal accounting strategies for a range of processing plants

Type of Operation	Examples and Recommended Strategies
Type 1 • High-grade/high-value feed • Medium flow rates • High-value product • Significant plant inventory • Weigh and sample all inputs: check in • Weigh and sample all outputs: check out	**Smelter, Metal Refinery** The recommended strategy is check in–check out.
Type 2 • Low-value feed—medium to high flow rates, often of coarse feed • High-value product • Low-mass, high-value plant inventory • Weigh and sample product	**Gold Operation** The recommended strategy is to reconstitute tailings at measured feed rate and production.
Type 3 • Low-value feed—medium to high flow rates—sampled after primary grind • Low-value product—sampled as concentrate shipments • Low plant inventory • Conveyor weigh scale or weigh feeder for feed mass into plant • Accurate sampling of fine tailings, feed analyses, and tailings analyses at measured feed rate and production rates	**Base Metal Concentrator** The recommended strategy is to mass-balance feed analyses, concentrate, and tailings.
Type 4 • High tonnage of feed and products • Minor rejects; low plant inventory • Sometimes direct shipment to customer • Detailed product specifications • International standards for sampling and characterization • Custody transfer mass will often be a draught survey or a train loadout weight	**Coal Operation, Iron Ore** A commodity sales contract will detail prices and penalties as well as acceptable measurement techniques for both producer and buyer. The recommended strategy is to weigh and sample feed and product streams.
Type 5 • High-value, low-tonnage feed • High-value products • Minor reject streams; significant plant inventory • Weigh and sample all inputs: check in • Weigh and sample all outputs: check out	**Precious Metals Refineries** The recommended strategy is check in–check out.
Type 6 • Low-value, high-tonnage feed • Minor rejects, usually based on particle size or density • Minimal plant inventory	**Aggregates** A commodity sales contract will detail prices, usually based on tonnage and size specification. The recommended strategy is to weigh feed and product streams.
Type 7 • Low-grade, high-tonnage feed • Residues remain in situ in some cases • Very high, difficult-to-measure process inventory • Non-steady-state, two-phase, slow reaction kinetics	**Heap Leach** Weigh, sample, and analyze feed and product. Perform accurate solution balance. The recommended strategy is to perform periodic checks of heap solution inventories and heap residue metal contents.
Type 8 • Medium- to low-value, high-tonnage feed • Low process inventory • Requires use of mineralogical analysis • Products may be in bulk, bags, or other containers	**Industrial Minerals** May be treated as a commodity. May be produced as different grades. The recommended strategy is to weigh feed and product streams.

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that, provided the errors are reasonably small (i.e., less than a few percent relative), the variance of a calculated variable is simply the sum of the products of the variance of each input variable and their partial derivative (with respect to each variable of interest) squared.

As most of the inputs are either fixed or linear, these derivatives are typically 0 or 1, and the error estimation process is much simpler than it sounds. Cutler and Eksteen (2006) and Morrison (2008) provide several examples, as do most texts on introductory statistics. Alternatively, a simple Monte Carlo analysis can provide indicative results as long as the error distributions for integrated assays and masses are reasonably realistic.

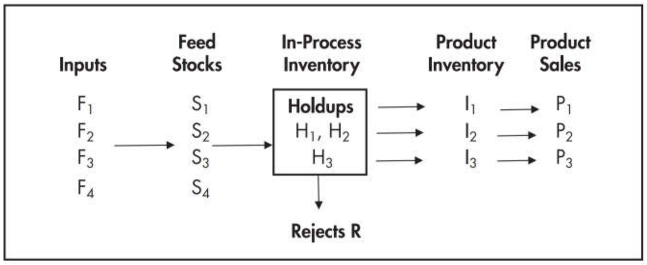
As a rule of thumb, the larger the assay differences achieved by the separation processes, the better the split factor will be defined. Hence, the two product mass and metal split estimates across a complete plant should be more accurate than any similar calculation for each single stage of separation.

Case Study 1: Northparkes Concentrator Example (adapted from Jansen et al. 2007)

Figure 1 shows the metal accounting flow sheet in use at an Australian copper–gold concentrator. Northparkes was the site nominated by Rio Tinto as sponsors of the code's development. Because of sponsor involvement, statistical errors in the measurement and accounting process could also be measured.

The Northparkes concentrator has two SAG/ball grinding circuits with flash flotation followed by a conventional flotation plant. The concentrator is unusual in that it has two processing lines, one of which is much larger in capacity than the other.

For a more detailed description, see Jansen et al. (2007) and Butcher et al. (2013). Figure 1 shows the key measurement



Source: Morrison 2008

Figure 4 Generic metal accounting flow sheet for a typical mineral processing plant

Table 2 Data usage and cut frequencies for on-stream analyzer samples

Slurry Stream	Process Control	Metallurgical Accounting	Cut Frequency, minutes Module 1	Module 2
Flotation plant feed	X	X	30	30
Flash flotation concentrate	X	X	60	20
Final concentrate (combined)	X	X	40	15
Final tailings	X	X	10	40
Rougher tailings	X	—	—	—
Cleaner scavenger feed	X	—	—	—

Source: Jansen et al. 2007, reprinted with permission from the Australasian Institute of Mining and Metallurgy

points. The points at which custody transfer occurs should be fairly obvious. External transfers occur from mine to concentrator at the stockpile feed and of the filtered concentrate to the rail-head silos.

The work reported by Jansen et al. (2007) was carried out under the AMIRA P754 project. The accuracies of almost all of the inputs to metal accounting were investigated. The concentrate weightometers were calibrated with static weights supported by the weigh frame, and conveyor speed was checked by multiple direct measurements of time of travel for 40 m of feed belt.

The weigh-frame calibration resulted in a correction of 3%, but the conveyor speed tests indicated that the speed measurement was both precise and accurate. Some concentrate buildup on the conveyor weigh frame might have contributed to the required adjustment.

Moisture

Northparkes used periodic samples of 3–5 m of SAG mill feed to obtain measurements of feed moisture. This is a labor-intensive process and an average moisture of 2% was the result of this testing. Hence, 2% was used as a factor thereafter. Although this assumption has little effect on the variance of the dry tonnage estimate, it could easily lead to a bias in the totalized feed measurement.

A much finer feed material and regular crossbelt sampling make concentrate conveyors a much more tractable target. Average measurements of 7.5%–8.5% moisture achieved relative standard deviations of less than 1.5%.

Mass Balance

The key mass balance is around the flotation plant. It resembles but is not exactly a two-product balance because the flash flotation cells treating cyclone underflow bring the grinding circuit into the balance. Key streams are sampled by pressure pipe samplers and directed to the onstream analysis system for process control. A subset of samples is used for metallurgical accounting (see Table 2).

Jansen et al. (2007) investigated the sampling variances and assaying variances for samples taken immediately after one another to minimize the impact of process variation. The reproducibility of the sampling process was 4% to 8% (relative standard deviation) for sample mass per increment.

Given that many samples were taken per accounting period of 24 hours, any effect on a composite sample should be small. Because the standard deviations of assays across the entire circuit were measured, Jansen et al. (2007) could develop an error model for copper assays, including sampling. Figure 5 shows that a relative error model with a small offset is appropriate for absolute standard deviations of copper assays. The error model shown in Figure 5 provides an estimate of the expected Cu standard deviation for any stream in the circuit.

Error models can significantly improve the accuracy of the balancing process because they bring quantitative process experience into the balance. Conversely, unrealistic error estimates can lead to inappropriate balances.

CALCULATIONS AND REPORTABLE FIGURES

The true product of any metal accounting system is a series of reports. These reports will vary in the time spans they cover and the level of management they are prepared for. What they have in common is that they should be sufficiently reliable to support sound decisions. Figure 6 shows a section of a typical monthly production report.

The Northparkes system uses a "back to front" strategy because of the difficulty of sampling –200 mm SAG feed. This approach is made more complicated by the flash flotation circuit. Without that circuit, the two-product approach could be applied directly.

Starting with an overall dry tonnage balance for the period:

$$F = C + T$$
feed = concentrate + tailings

The measured values of feed and concentrate are "wet." Therefore, some factors (designated by Q) are applied based on an arbitrary sampling series to obtain realistic values for sample moisture:

$$F = Fwet - (Fwet*Qfeed)$$
$$C = Cwet - (Cwet*Qcon)$$

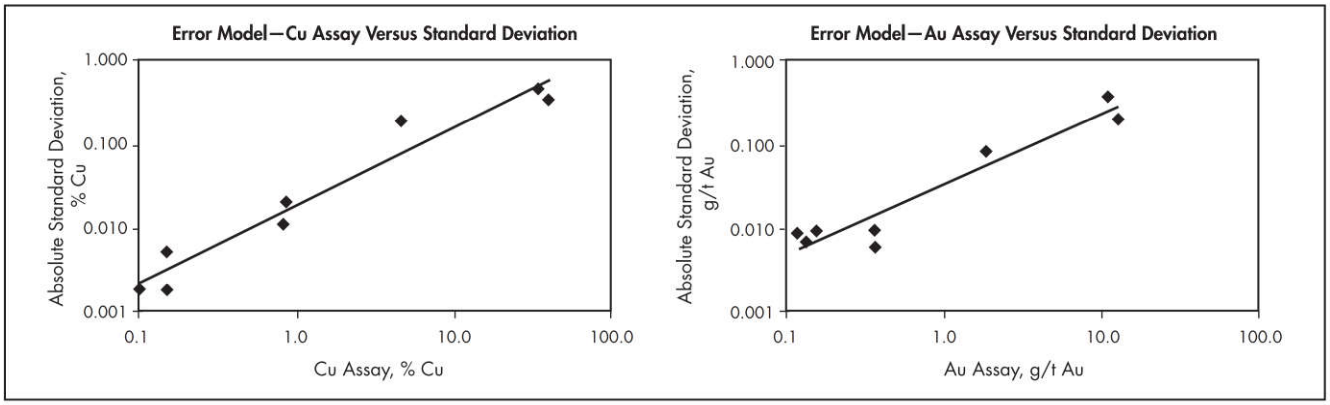
For each element (designated by lowercase letters) around the complete module:

$$Ff = Cc + Tt$$

where lowercase letters indicate an assay in that stream. The assays that are important are for copper and gold.

Recalling that the feed assays are not available but they can be directly calculated:

$$f = (Cc + Tt)/F$$



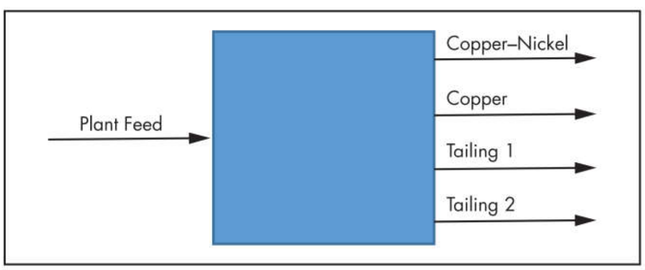
Source: Jansen et al. 2007, reprinted with permission from the Australasian Institute of Mining and Metallurgy

Figure 5 Error models for copper and gold assays, including sampling

Monthly Production Statistics—Ore Processing				
Parameter	**Unit**	**Module 1**	**Module 2**	**Total**
SAG Mill Feed	dmt	129,831	202,631	332,462
Feed Grade	Cu%	0.83	1.08	0.98
	Au g/t	0.52	0.37	0.43
Concentrate Grade	Cu %	32.89	36.17	35.02
	Au g/t	17.26	9.81	12.42
Recovery	Cu%	91.12	91.49	91.37
	Au%	76.76	71.6	74.03

Source: Jansen et al. 2007, reprinted with permission from the Australasian Institute of Mining and Metallurgy

Figure 6 Excerpt from a typical metal accounting monthly report



Source: Lachance et al. 2012, reprinted with permission from the Canadian Institute of Mining, Metallurgy and Petroleum

Figure 7 Strathcona flotation plant primary metallurgical accounting flow sheet

Similarly for recovery, R (%), in each element of interest by definition:

$$R = 100^* (Cc/Ff)$$

However, recovery can also be estimated from the assays:

$$R = 100^*[c(f - t)/f(c - t)]$$

The last reportable amount, M, is the metal contained in the concentrate:

$$M = Cc$$

for either copper or gold.

As for the generic balance, the rule for propagation of error can provide some error bounds for calculated variables. Error models can provide very useful input to this calculation.

Perhaps unsurprisingly, this error analysis comes to similar conclusions to one based on three measured products, except that the feed grade is estimated. The feed-grade estimate is most sensitive to errors in the wet concentrate measurement. Because that value is part of the feed calculation, it also dominates recovery errors where the tailings error would usually dominate.

Case Study 2: The Strathcona Mill

Lachance et al. (2012) provide an excellent case study of a two-stage update of the metal accounting system at Xstrata's Strathcona mill from a four-product balance to a code-compliant system. This case study summarizes some key aspects of that paper.

The Strathcona mill is a complex operation treating copper–nickel ores from Xstrata and non-Xstrata sources. Some of these ores contain significant platinum group metals. For metal accounting purposes, the concentrator can be considered as a single processing block with one feed, copper–nickel concentrate, copper concentrate, and two tailings streams (Figure 7). In the original system, three assays (nickel, copper, and sulfur) were used in a so-called four-product formula (Hodouin et al. 2011). Clearly, if only three assays are available, the various flow splits are singly determined. However, as many stages of separation of the minerals (which are summarized by their assays) have occurred within the accounting block, the flow splits may still be reasonably well determined.

As a general rule, the mass split across the entire concentrator is usually better defined than for single stages of separation because the differences between feed, products, and rejects assays are larger compared both in absolute terms and in comparison with measurement errors. However, the lack of redundancy in this example means that there is little chance of error detection within those flow splits. The only mass flow measured was the feed to the mill. The feed stream was sampled for assay and for moisture measurement.

The system upgrade amounted to an audit based on the principles of the draft code (AMIRA 2007). To achieve compliance, it was necessary to replace a spreadsheet-based system with a relational database and a well-controlled mass balancing environment for reconciliation. For this application, the team applied a commercial system (Metallurgical Account, which uses the well-documented BILMAT algorithm). These changes increased integration and provided a much-improved degree of compliance with the code.

The code strongly recommends against spreadsheets, both for data analysis and for data storage, because they provide an environment with extreme ease of editing and one in which it is almost impossible to maintain integrity of either data or calculations (AMIRA 2007). The elemental assay suite was increased in two stages to a total of 13. This number of assays provided a strong degree of redundancy, which should much improve error detection. LaChance et al. (2012) concluded after a trial period that the addition of Fe, MgO, and SiO_2 assays provided the best benefit to balance in terms of accuracy.

Overall, this paper provides a practical strategy for upgrading a "traditional" metal accounting system to one that is strongly compliant with the draft code of practice. However, it is also worth noting that retaining the block separation approach does not expose opportunities for enhanced long-term process understanding. That is, it does not improve opportunities to add value through metal accounting. If that has not already been done, it might provide a third-stage upgrade for the Strathcona operation.

CONCLUSIONS

Metal accounting based on the draft code of practice (AMIRA 2007) provides a structured approach to development, management, and audit of credible systems across a wide range of types of mineral processing. It also offers strategies for detection of short-term errors and long-term bias.

Achieving a single set of well-supported flow rates and assays—with error bounds—should greatly simplify decision making based on performance-related claims and decisions.

The draft code is most strongly focused on performance of the processing plant. However, there will almost always be further benefits in extending this approach to resource to disposal accounting and to balancing around each section within a processing plant.

For those who believe this might entail too much time and effort, it is worth remembering that not losing a kilogram of metal is appreciably more profitable than having to mine and process additional ore to replace it.

ACKNOWLEDGMENTS

This chapter, the draft Code of Practice and Guidelines, and the metal accounting textbook are all outcomes of AMIRA P754. The author acknowledges the contributions from the industry sponsors of P754, the Code Team, and the researchers and students from the universities of Cape Town, Stellenbosch, and Queensland. The AMIRA project manager, Richard Beck, and the leader of the Code Team, Peter Gaylard, deserve special mention for their outstanding contributions.

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