CHAPTER 7.9

Applied Flotation Modeling

Peter Amelunxen and Richard LaDouceur

Froth flotation models are useful for many purposes, and each purpose often requires a unique set of model characteristics. As such, different modeling approaches have evolved to fit different purposes or objectives, and sometimes, they are not interchangeable. For example, a simulator developed to populate a block model with plant performance parameters will not be very useful for helping a cell manufacturer develop a more efficient cell mechanism. A dynamic flotation model, developed to predict flow surges and improve control loop response in real time, is not likely to be the primary tool for flow sheet development, which requires detailed ore characterization and some kind of economic optimization strategy. Given the diversity of modeling approaches, it was decided that this chapter, being part of an engineering handbook, should focus on the most common *applied* flotation models and their components, that is, those used for steady-state simulation, flotation circuit design, process optimization, and geometallurgy. Where appropriate, the reader is referred to sources covering alternative applications.

BACKGROUND

The practice of mathematical modeling far predates the development of the flotation process, and therefore, in many respects, the history of flotation modeling is a history of flotation itself. P.A. Amelunxen and Runge (2014) trace that history through the past 100 years or so, identifying several key developments in the field. Some were spawned by developments in our own field of extractive metallurgy; for example, the adoption of compartmental models was probably driven, in large part, by the commercial implementation of column flotation in the 1980s. Other milestones were achieved through advances outside mineral processing. Probably the single most important event in the history of flotation modeling is the widespread adoption of personal computers during the early 1990s, an event that gave researchers access to new tools to study flotation machines and gave plant designers and operators access to more advanced flotation simulation tools. Although it is true that many of the components of flotation models have been known for much longer (e.g., first-order kinetics models, hydraulic entrainment, and froth recovery have been recognized since at least the early 1960s), the packaging of these concepts into easily accessible and reliable simulation software is relatively recent.

When reading this chapter, we encourage the reader to remember that there is no magic involved in modeling froth flotation. Anybody can build a flotation model or simulator using a laptop, a spreadsheet, and a simple scripting or macro language (to handle iterations and circulating load calculations).

OBJECTIVES AND VALUE OF FLOTATION MODELS

There can be multiple objectives for flotation modeling and simulation. As with any engineering discipline, the objectives are focused on value creation:

- **Process design** reduces the cost and time requirements or decreases the risks associated with flotation process design, engineering, and/or construction projects.
- **Process optimization** increases the recovery of valuable minerals, improves the separation, reduces the costs, or increases the revenues of an existing flotation process or system.
- **Process control** improves overall circuit performance through online process control and optimization, usually as part of a broader advanced process control strategy and system (Shean and Cilliers 2011).
- **Geometallurgy** evaluates the susceptibility of a flotation circuit to changing ore characteristics, such as feed grades, mineralogy, liberation, and other parameters, and determines how this variability affects the economic return of a given flotation process or system.

There can be little doubt that flotation simulators over the past two decades have contributed significantly to the field of mineral processing (Lane et al. 2005). In the authors' experience, this is evidenced by the employment of simulators in the design and optimization of virtually every large, contemporary mineral flotation process.

Peter Amelunxen, President, Aminpro, Lima, Peru
Richard LaDouceur, Postdoctoral Research Scholar, Montana Tech, Butte, Montana, USA

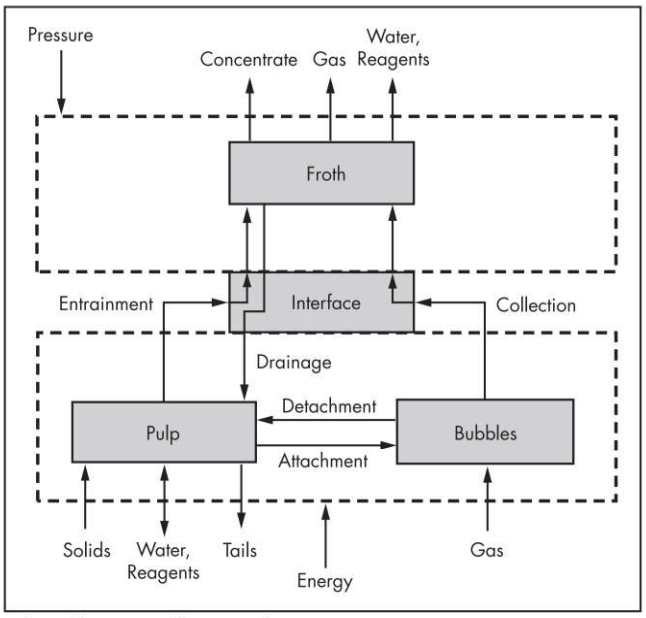
THE FLOTATION SYSTEM

A froth flotation cell can be described as an open system because there is mass and energy exchange with the environment. The inputs include solids (feed), water, reagents, energy, and gas. The outputs include solids (concentrate and tails), water, energy (heat, noise), reagents, and gas. The system is also subject to atmospheric pressure, the magnitude of which depends on the design of the cell (pressurized, nonpressurized) and the elevation above sea level of the system. The system has several components, including a pulp zone, froth zone, and pulp–froth interface zone, and three phases (solids, water, and gas). For modeling purposes, de-inking, de-oiling, and other pseudo-liquid recovery applications are considered to be three-phase systems.

The system has several mass and energy transfer mechanisms as depicted in Figure 1, and some of these are interrelated and dynamic by nature. It is the purpose of a flotation model to provide a simplified mathematical framework to describe this complex system such that the objectives listed earlier can be met. The following "Flotation Machine Models" section describes the commonly used mathematical framework used to represent flotation systems.

Figure 1 clearly shows that the flotation system is complex, and even today, some of the underlying mass and energy transfer phenomena can be neither completely characterized nor fully understood. These gaps in our knowledge must be filled with either an assumption or a simplification, and, as a result, all flotation models and simulators contain some element of empiricism. This becomes extremely important when performing modeling and scale-up from laboratory flotation tests. It is precisely how these small elements of empiricism are handled that drives the confidence in the resulting model or scale-up calculation, and it is for this reason that one cannot discuss flotation modeling without also discussing laboratory test methodologies. This is presented later in the "Laboratory Testing" section of this chapter.

It is important to distinguish between a flotation model and a flotation simulator. A flotation model is a mathematical framework, while a flotation simulator is a software tool that contains models. Once the modeling framework and the laboratory test procedures have been determined, a flotation simulator can be chosen or constructed that has the necessary functionality for the job at hand. This functionality and some commonly available simulation approaches are described later in the "Simulators" section.



Adapted from P. Amelunxen et al. 2018

Figure 1 The flotation system

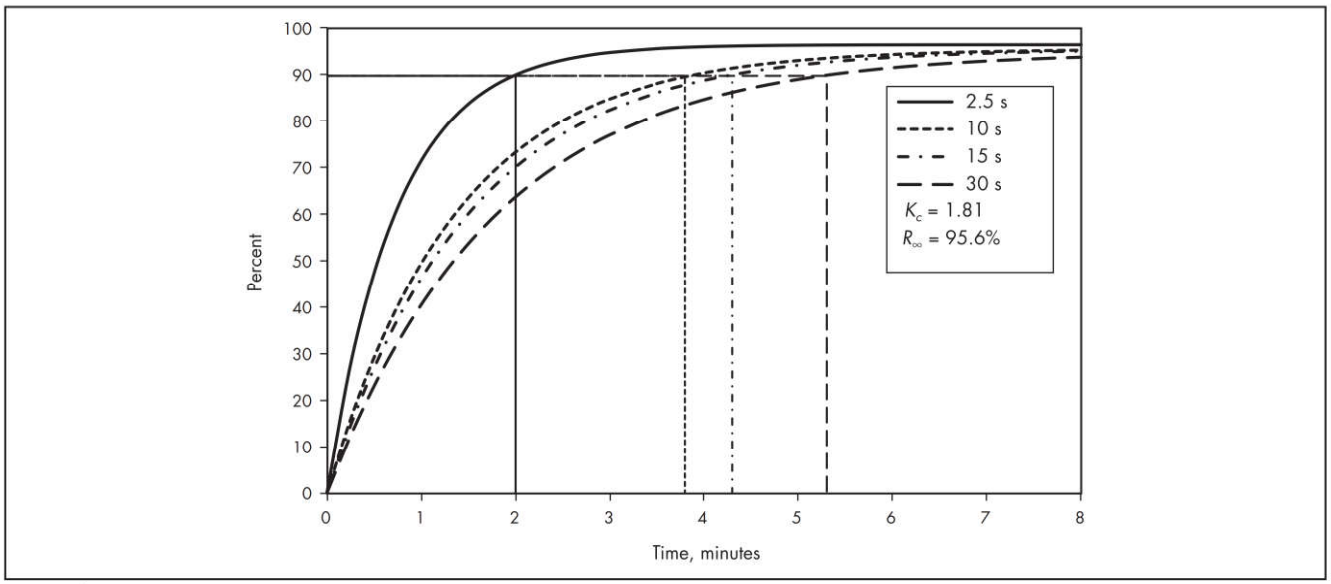
FLOTATION MACHINE MODELS

Mathematical models of continuous flotation machines can be divided into three classes: empirical models, phenomenological models, and physics-based models. Empirical models are relatively simple models that rely on an individual's experience or a set of industrial benchmark data to convert laboratory test results into industrial plant performance estimates. Phenomenological models seek to mathematically model the underlying rate processes or mass transfer using data gathered at the laboratory scale to predict industrial plant performance. Physics-based models seek to replicate the fundamental particle/fluid dynamics of the system by discretizing it into small individual elements, calculating the mass and energy balance for each element, and summing the elements to determine the performance of the whole. The method relies on a rigorous understanding of the interparticle and multi-fluid systems (a combination of discrete element methods and computational fluid dynamics), an appropriate programming shell to code that understanding, and sufficient computational processing power to simulate it. These models are often proprietary and are not used for designing circuits (yet). They are not discussed in this chapter.

Empirical Models

Empirical models originated during the 1950s, when test procedures focused on determining the flotation kinetics mainly for those elements that would generate value for the project; little was done to characterize the floatability of other minerals, such as pyrite and gangue, which provided no income. Because an accepted method for scaling up the results from a laboratory kinetics test to commercial level did not yet exist, many engineers developed their own methods from personal experience. One of the most common was the use of flotation time factors. For example, Lindgren and Broman (1976) wrote that the plant flotation time should be estimated by applying a factor of two to the laboratory time at the point where the cumulative recovery leveled off, while others recommend factors between 1.6 and 2.6 (Dunne et al. 2002). Others define the point at which the recovery levels off as the point at which the incremental concentrate grade drops below the feed grade.

There are several drawbacks to these kinds of approaches. One drawback is that extensive experience and benchmarking is required with the particular flotation application under design. As a result, different engineers, with different backgrounds and data, will arrive at different sets of design criteria. For smaller, simpler applications, this may not be a significant obstacle, because the engineer or project owner can correct for risk through engineering margins and safety factors—usually by adding some extra flotation time at the end of the bank, increasing pump and launder capacities, and so forth. However, for modern, low-grade, mass mining projects, the capital cost of these design margins and safety factors can be quite substantial, and the cost of under-designing the flotation circuit even more so.



Adapted from P. Amelunxen et al. 2018

Figure 2 Recovery versus time for four scraping rates

Another drawback is that because each laboratory technician might scrape the froth at his or her characteristic speed, reproducibility of the test results is often relatively poor. Differences in water quality, froth and frother characteristics, and air flow rates also contribute to error. This can be seen in Figure 2, which shows the experimental recovery versus time curve for a copper mineral, for four scraping rates (2.5, 10, 15, and 30 seconds). Each curve is the average of triplicate tests (P.A. Amelunxen et al. 2014). Assuming a 2.5× scale-up factor for these conditions, the resulting continuous industrial bank retention times would be, respectively, 5, 9.5, 10.6, and 13 minutes.

It is mainly because of these drawbacks, combined with better understanding and accessibility of phenomenological models, that empirical models are usually only used for scoping or cursory level testing, or for applications that require low costs, such as geometallurgy.

Phenomenological Models

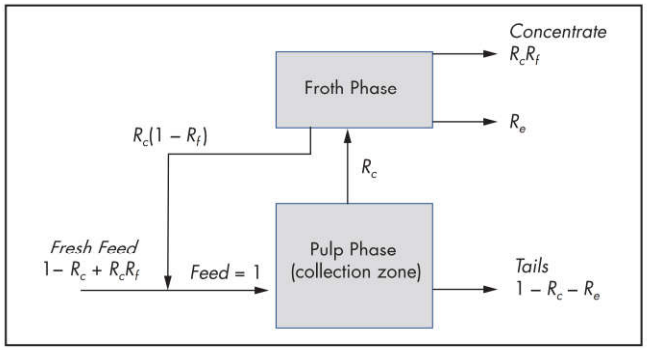
Because of the disadvantages of empirical models as discussed previously, large-scale flotation projects increasingly rely on phenomenological modeling methods for process design and optimization. The key components of phenomenological models are described in this section.

Mechanical Flotation Cells

Mechanical flotation machines are modeled using the so-called compartmental model of flotation, in which the various mass and energy transfer mechanisms shown in Figure 1 are simplified into two separate compartments or submodels; these are shown by the dashed line in Figure 1, and are

- The *collection zone*, where mineral is collected onto bubbles in the pulp, and
- The *froth zone*, where the concentrate is transported to the lip of the cell.

Figure 3 shows the graphical depiction of the compartmental model, including the relevant mass transfer vectors.



Adapted from Finch and Dobby 1990

Figure 3 Compartmental model (entrainment not shown)

The total recovery of a given mineral class, R, is the sum of the material recovered by entrainment and by collection, minus drainage.

If the fractional froth recovery, R_f, is defined as 1 minus the drainage, we can derive the so-called compartmental model of flotation (a better term is probably *dual-compartment model*). This is given by the following equation (Falutsu and Dobby 1989):

$$R = \frac{R_c R_f R_\infty}{R_c R_f + 1 - R_c} + R_e \qquad \textbf{(EQ 1)}$$

where R_c is the collection recovery, R_∞ is the maximum recovery at infinite time (often defined by mineral size or liberation effects), and R_e is the entrainment recovery.

Collection zone. The collection recovery, R_c, of a given mineral particle class is usually expressed as a first-order rate process relative to time. For plug flow and continuous flow, the equations are, respectively,

$$R_c = 1 - e^{-kt} \qquad \textbf{(EQ 2a)}$$

$$R_c = \frac{k\tau}{1 + k\tau} \qquad \textbf{(EQ 2b)}$$

where t is the residence time in a plug flow reactor, τ is the mean residence time in a continuous perfectly mixed reactor, and k is the overall pseudo first-order rate constant of a given hydrophobic particle class, with respect to time (Garcia-Zuñiga 1935; Arbiter and Harris 1962; Dorenfeld 1962; Levenspiel 1962).

Once the cumulative recovery versus time profile is derived from the laboratory batch kinetics test, the first-order kinetic rate constant can be calculated independently for each mineral using curve-fitting techniques. There are three common methods for doing this. The most common approach is simply to apply Equation 2a directly to the time versus recovery profile, in which case, it is implied that entrainment in a laboratory test is zero while froth recovery is 100%. A second approach is to account for froth recovery and entrainment by combining Equation 2a with Equation 1 and solving for k, R_∞, R_f, and R_e for all minerals simultaneously using curve-fitting techniques (P.A. Amelunxen and Amelunxen 2009). A third approach involves calculating R_f and R_e from hindered particle settling theory and a water balance around the froth zone, and then solving for k and R_f independently for each mineral (P. Amelunxen et al. 2018).

Distributed rate constants. Equations 2a and 2b describe the classical approach, in which each mineral or element is assigned a single rate constant. It has been shown that the value of k for a given system is a function of the key parameters that define that system:

- Attachment and detachment rates (Schuhmann 1942)
- Feed characteristics, such as particle size (Gaudin et al. 1942; Morris 1952)
- Mineralogy (including particle oxidation), mineral association, and mineral surface expression (Imaizumi and Inoue 1965)
- Operating conditions such as reagent dosage or reagent surface coverage (Schuhmann 1942)
- Machine-dependent characteristics such as bubble saturation or impeller energy input (Sutherland 1948)

From a philosophical perspective, each individual particle would deserve its own rate constant and maximum recovery, depending on its size, shape, density, mineral constituents, hydrophobic surface expression, and hydrophobicity. This becomes mathematically unwieldy, and researchers have sought to reduce the complexity by expressing the distribution of particle species in a simplified form. The species distribution could be assumed to be discrete or continuous. Some proposed forms of discrete distributions include

- Single rate constant methods (classical) (Garcia-Zuñiga 1935; Thorne et al. 1965),
- Dual rate constant methods (Klimpel 1980), and
- Particle-size-based rate constants (Jowett 1974).

Some proposed forms of continuous distributions include

- Rectangular distribution (Huber-Panu et al. 1976; Klimpel 1980),
- Triangular distribution (Harris and Chakravarti 1970),
- Normal distribution (Chander and Polat 1994),
- Sinusoidal distribution (Diao et al. 1992),
- Weibull distribution (Dobby and Savassi 2005),
- Particle-size-based distribution (Morris 1952; Ofori et al. 2014; P.A. Amelunxen and Amelunxen 2009), and
- Gamma distribution (Imaizumi and Inoue 1965; Yianatos et al. 2010a).

The strengths and weaknesses of each of the preceding approaches have been discussed at length in the literature (Dowling et al. 1985; Polat and Chander 2000); this discussion is not repeated here. It is worth noting, however, that from the preceding list, there are three more common industrially and commercially applied methods:

1. The dual rate constant method, which forms the basis of JKTech's SimFloat simulator (Runge 2007) among others;
2. The Weibull distribution, which forms the basis of SGS's integrated geometallurgical simulation (iGS) flotation simulator (Dobby and Savassi 2005) among others; and
3. The particle size-based method, which forms the basis of Aminpro's AminFloat simulator (R.L. Amelunxen and Amelunxen 2009) among others.

Machine-dependent parameters. It has long been recognized that the characteristics of the flotation machine influence the value of the rate constant. Some practitioners vary the rate constant with bubble surface area flux (S_b). Bubble surface area flux is the superficial rise rate of bubble surface area per cross-sectional area. For a given flotation machine, this parameter can be directly related to flotation cell operating parameters and drives flotation rate (Gorain et al. 1998b):

$$k_c = PS_b \quad \textbf{(EQ 3)}$$

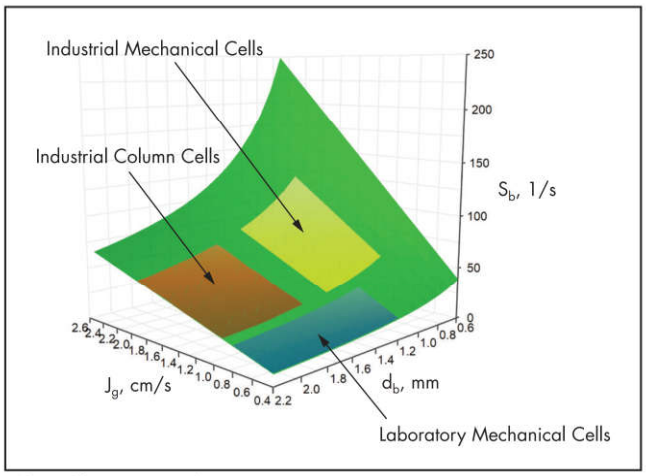
where k_c is the collection zone rate constant with respect to time, and P is the proportionality constant (often termed the *ore floatability* or *floatability index* [Collins et al. 2009]). The bubble surface area flux is directly related to bubble size and the superficial gas velocity (Finch and Dobby 1990):

$$S_b = \frac{6J_g}{d_{32}} \quad \textbf{(EQ 4)}$$

where J_g is the superficial gas velocity and d_{32} is the Sauter mean bubble size diameter. The importance of Equations 3 and 4 for interpreting laboratory kinetics tests and applying the results to simulating full-scale machine performance should not be underestimated. This can be seen in Figure 4, which shows the S_b plotted against gas velocity and bubble size for different flotation machines (laboratory, industrial mechanical cells, and industrial column cells). There can be a very large difference between the S_b of a laboratory cell and the S_b of an industrial cell, and applying Equation 3 may result in as much as a 400% adjustment to the observed collection rate scaling from laboratory to plant. For brownfield applications (plant optimization and some geometallurgy programs), the issue can be resolved by performing plant audits and validating the input parameters used for the scale-up procedure. For greenfields or other design applications, the plant S_b must be estimated from benchmark data and then fine-tuned (via impeller speed, impeller geometry, or frother strength) after the plant has been commissioned.

Although the preceding method is a common approach to incorporating machine influence, other methods exist. One alternative, reported by Yianatos et al. (2010b), is to multiply the laboratory rate constant by a factor, usually based on experience or benchmarking, to account for the machine-dependent parameters such as froth effects, mixing effects, and particle segregation effects.

Entrainment. Particles are also recovered through hydraulic entrainment, caused by viscous drag forces acting on the particle surface as water, froth, and collected solids



Adapted from Vera et al. 2002

Figure 4 Comparison of gas distribution parameters between laboratory, industrial mechanical, and column flotation cell

are recovered to the concentrate. When the particle settling velocity in the presence of these drag forces is lower than the net rise velocity of water, the particle will be entrained to the concentrate. Many factors affect the rate of hydraulic entrainment, including solids percentage; particle size, shape, and density; cell turbulence, particularly at the interface; froth height; gas flow rate; froth residence time; rheology; frother type and concentration; particle hydrophobicity; and concentration of hydrophobic particles (Wang 2016). The entrainment recovery of solids, R_e, is the percent of a given size class that is recovered by entrainment, and many mathematical models have been proposed to describe R_e as a function of the various system parameters (Bisshop 1974; Bisshop and White 1976; Moys 1978; Ross and Van Deventer 1988; Maachar and Dobby 1992; Savassi 1998; Neethling and Cilliers 2002, 2009; Yianatos and Contreras 2010). A common approach is to assume that for a given size class, R_e is proportional to water recovery (R_w) (Johnson et al. 1974; Smith and Warren 1989). Two commonly applied models are (Jowett 1966; Engelbrecht and Woodburn 1975; Laplante et al. 1989) shown in Equations 5a and 5b:

$$R_{e,i} = \frac{ENT_i R_w}{1 + R_w(ENT_i - 1)} \quad \textbf{(EQ 5a)}$$

and its simplified form

$$R_{e,i} = ENT_i R_w \quad \textbf{(EQ 5b)}$$

where ENT_i is a proportionality constant, also termed the *degree of entrainment*, and i is the particle size class.

These equations are simplifications and may break down near boundary conditions, such as at high R_w; very coarse particles; and high turbulence, particularly at the interface. Also, R_e is usually applied to gangue particles, but it should also be applied to uncollected hydrophobic particles. By convention, collection is calculated first and entrainment is applied to particles remaining after collection.

Water recovery. The water recovery of industrial cells is difficult to estimate from laboratory data without detailed knowledge of the machine and pulp characteristics—knowledge that is difficult to obtain for greenfield applications. For this reason, water recovery is one of the few parameters that is still often determined based on experience or empiricism (another is froth recovery). Commercially available models offer up to four methods for modeling water recovery in industrial cells (JKTech 2004):

1. **The concentrate percent solids set-point method.** The user specifies the mass fraction of solids in the concentrate slurry (not including air) and water recovery is back-calculated from the feed water flow rate.
2. **The first-order rate process method.** The user specifies a rate constant and water recovery is calculated from the corresponding first-order rate equation (plug flow or perfect mixer; in this case, froth recovery is ignored).
3. **The froth residence time method.** Water recovery is (usually linearly) proportional to the froth residence time, which is approximated from the froth height, air rate, and cell dimensions (the proportionality constant or constants are user-specified and must be derived from experience or plant surveys).
4. **The constant water recovery method.** The user simply specifies a fixed value for the water recovery.

Because of the relatively high water recovery that occurs in a laboratory kinetics test (relative to a continuous industrial cell), entrainment phenomena are often quite significant at the laboratory scale, and should, therefore, be accounted for when interpreting laboratory flotation kinetics tests. In this case, water recovery is known from a water balance derived from mass measurements on the feed and product stream. The effect of entrainment of laboratory rate constants is usually negligible for coarse, fast-floating minerals but can be very significant for fine, slow-floating minerals (examples include pyrite separation in reground intermediate concentrates and molybdenite separation from reground copper sulfides).

Froth recovery. Froth recovery should not be confused with air recovery or bubble recovery. It is specific to a given mineral class and is defined as the fraction of a mineral class recovered by collection that is subsequently recovered to the concentrate. If all of the collected mineral is recovered, the froth recovery is equal to unity ($R_f = 1$). If none of the collected mineral class is recovered, such as when a cell is not overflowing froth, then the $R_f = 0$. The froth recovery is driven by the operating and machine characteristics of the industrial cell. The following lists some of the recognized relationships between the froth recovery and the cell operating conditions:

- Higher air rates can lead to higher froth recovery, up to a peak (Hadler and Cilliers 2009).
- Deeper froth beds can lead to lower froth recovery (Feteris et al. 1987; Vera et al. 2002).
- Higher ore grades, higher fines concentration, and higher hydrophobicity can stabilize the froth film, and therefore lead to higher froth recovery (Morris et al. 2008).
- The type and concentration of frothers (Cho and Laskowski 2002) or electrolytes (Wang 2012; Biçak et al. 2012) can impact the froth stability, with higher concentrations leading to higher froth recovery.
- Shorter froth residence times, such as those created by internal launders or mechanical scrapers, lead to higher froth recoveries (Contreras et al. 2013).
- In some cases, extremely high carrying rates lead to lower froth recoveries, such as when the cell's carrying capacity per unit area is approached (Patwardhan and Honaker 2000).

It is difficult to accurately measure the froth recovery of a given system, and there are no predictive models that work well for greenfield design applications, although some froth models have been reported to be useful for process control (Neethling et al. 2006). For this reason, froth recovery is usually a user input and is based on experience. When flotation circuit operators vary the pulp level and air rate to a flotation cell, froth recovery and water recovery change, and for this reason, most commercial simulators link the froth recovery to the froth depth and superficial gas velocity (air rate may also be linked to collection recovery, per Equations 3 and 4). Some models relate the froth recovery directly to the froth depth and air rate; others to the froth residence time (Gorain et al. 1998a), which requires knowledge of the cross-sectional geometry of the cell, including froth crowder, concentrate launder, and lip length geometry.

Mechanical constraints. The two main mechanical constraints that are considered for process design are carrying capacity (*Ca*) and lip loading capacity. They apply to all machine types.

Carrying capacity is expressed in units of metric tons per hour of solids that may be recovered per square meter of cross-sectional area of the cell. It can be modeled empirically, using either the method of Finch and Dobby (1990),

$$C_a = K_1 \pi d_p \rho_p \left(J_g^{(1-q)}\right) \quad \textbf{(EQ 6)}$$

or the method of Patwardhan and Honaker (2000):

$$C_a = 522.54 d_{50}^{-0.542} \sigma^{0.2} \frac{n_p d_{50}^3 \rho_p}{d_b^3} J_g \quad \textbf{(EQ 7)}$$

where K_1 and q are fitted, ρ_p is the mineralized bubble density (in the aggregate), J_g is the superficial gas velocity in the cell, d_{50} is the 50% passing size, σ is the size modulus, and d_b is the bubble diameter. For detailed discussion of the usage and applicability of these empirical equations, the reader is referred to the cited reference.

Lip loading capacity is expressed in metric tons per hour of pulp (liquids and solids) that may be recovered per linear meter of lip length. It can be expressed as an empirical function of column diameter (R.L. Amelunxen 1991). The functions are mineral specific. Various methods are available to estimate pump, launder, and other slurry transport capacities; these are not discussed here.

Column flotation cells. The modeling of column flotation cells follows the same general framework presented for mechanical cells, with some key modifications. There are two more relevant modifications:

1. The collection zone recovery is modified to account for varying mean residence times between particle classes.
2. The entrainment recovery is modified to account for froth washing.

The first modification stems from recognition that the column cell may not be a perfectly mixed reactor, and therefore coarse particles may have higher internal settling velocities and shorter retention times than fine particles. There are several approaches; one commonly used is that described by Finch and Dobby (1990). The procedure involves several nonlinear interdependent formulas and requires a computational or iterative solution, as summarized in the following steps:

1. Start by assuming a value for slip velocity of a particle class relative to water, U_{sp}.
2. Calculate the particle Reynolds number, R_{ep}, for the mean particle diameter and the slip velocity using the following equation:

$$R_{ep} = \frac{d_p U_{sp} \rho_l (1 - \varepsilon_g)}{\mu_f} \quad \textbf{(EQ 8)}$$

where d_p is the mean particle diameter, ρ_l is the slurry specific gravity, ε_g is the fractional gas holdup, and μ_f is the fluid viscosity.

3. From the particle Reynolds number, recalculate the slip velocity from the following equation:

$$U_{sp} = \frac{g d_p^2 (\rho_p - \rho_{sl})(1 - \varepsilon_g)^{2.7}}{18 \mu_f \left(1 + 0.15 R_{ep}^{0.687}\right)} \quad \textbf{(EQ 9)}$$

4. Repeat steps 2 and 3 until the slip velocity converges.
5. Calculate the dispersion number, N_d, from the slip velocity using the following equation:

$$N_d = \frac{0.63 d_c \left(\frac{J_g}{1.6}\right)^{0.3}}{\left[\left(\frac{J_{sl}}{1 - \varepsilon_g}\right) + U_{sp}\right] H_c} \quad \textbf{(EQ 10)}$$

where d_c is the column diameter (or the diameter of the internal partitions, if the column is portioned), J_{sl} is the superficial slurry velocity, and H_c is the effective column height (i.e., the vertical distance from the spargers to the pulp–froth interface). The effective column surface area also changes with the number of partitions, and this must be considered.

6. Calculate the collection zone recovery from the following equation (also called the *Wehner–Wilhelm equation*) (Wehner and Wilhelm 1956):

$$R_c = 100\left(1 - \frac{4a e^{\frac{1}{2N_d}}}{(1 + a^2) e^{\left(\frac{a}{2N_d}\right)} - (1 - a^2) e^{\left(\frac{-a}{2N_d}\right)}}\right) \quad \textbf{(EQ 11)}$$

where a is defined as

$$a = \sqrt{1 + 4ktN_p} \quad \textbf{(EQ 12)}$$

7. Calculate the overall recovery from the compartmental model (Equation 1). Note that with wash water and positive bias (i.e., net flow of water downward), there is no recovery due to entrainment.

Equation 10 is the analytical solution of the mass transport differential equation that describes the concentration of reactants at a distance downstream from the point of injection in an axially mixed tubular reactor (i.e., no radial mixing) and ignoring the influence of nonuniform velocity profiles and short-circuiting. For plug flow conditions and continuous, perfectly mixed reactors, Equations 2a and 2b apply, respectively. For a more complete review of axially mixed reactor models applied to column flotation, refer to Tuteja et al. (1994).

Pneumatic Flotation Cells

There are currently no widely accepted phenomenological models for pneumatic cells, which is the class of flotation cells that includes Jameson cells, contact cells, and microcells. For this reason, most pneumatic flotation cells are scaled up from

pilot plant testing. If models are required (i.e., for geometallurgy or variability purposes), practitioners tend to adopt one or more concepts from mechanical or column cell models, and then use empirical corrections to account for the differences in kinetics between the two classes of equipment.

LABORATORY TESTING

Flotation models and simulators are often only useful when the model inputs—including the maximum recovery and kinetics information—are derived from laboratory test data and not industrial plant data. As such, it can be overlooked that the efficacy or fidelity of a given flotation model is closely linked to the procedures and precision of the laboratory test that is used to generate the model inputs. In this section, flotation test procedures are reviewed and discussed with reference to their impact on the resulting flotation models.

Test Methods

Simple Batch Tests

Simple batch tests are nonkinetic, single time interval tests with chemical or mineralogical characterization of the feed, tails, and one concentrate stream. They do not provide kinetics information and are usually only conducted for scoping or exploratory purposes, that is, to determine optimum pulp chemistry or grind conditions. The flotation time can vary and usually depends on the experience of the technician, the standards of the laboratory, and other considerations.

Kinetics Tests

The flotation kinetics test is the most important test for flotation circuit design and scale-up. The test is conducted by putting a known mass of sample in an agitated flotation cell with water and reagents and adding air and agitation for fixed time intervals, over which concentrate samples are collected. A curve of cumulative mineral recovery as a function of time is determined from the resulting feed, concentrate, and tails characterization. Once the curve of cumulative recovery versus time is derived, usually from a combination of quantitative mineralogical and stoichiometric information, the kinetics are calculated for each mineral using curve-fitting techniques. Unlike open cycle and locked cycle tests, kinetics tests can be used for process modeling and design. In fact, they are the only tests that should be used for phenomenological scale-up, as discussed in the next subsections.

Open Cycle Tests

Open cycle tests are performed by conducting a rougher batch test and then re-treating the product streams as required. Usually, the rougher concentrate is reground and refloated in a series of cleaning circuits. The tails from each stage are assayed at the end of the test, and the overall recovery is calculated through a variety of methods. The method usually depends on the assumptions made regarding any mineral values remaining in the unprocessed intermediate streams (sometimes termed the *hanging streams*). Open cycle tests cannot be used to derive the flotation kinetics and hence are not useful for circuit simulation.

Locked Cycle Tests

Like open cycle tests, locked cycle tests are composed of a series of batch flotation tests with or without intermediate regrinding of the concentrate. The tails, concentrates, and water of the various cleaner or scavenger stages are returned to the prior stage of the subsequent cycle (depending on the particular configuration of the test). In this manner, the closed-circuit configuration of an industrial plant is mimicked, although it is constructed from a series of batch tests rather than from a series of continuously fed flotation banks. These tests have several drawbacks; chief among them is that they do not provide kinetics information. For this reason, they are rarely used for simulation purposes.

Pilot Plants

Pilot plants share some of the disadvantages of locked cycle tests. The higher water recovery resulting from the shallower froth heights typical of pilot scale cells often leads to excessive dilution in the cleaners. This can impact the concentrate quality and the separation kinetics and create bias when extrapolating the performance to plant conditions. For this reason, phenomenological models are usually developed with batch kinetics tests and then, if required, validated using pilot plant tests. If the model predicts the mineral distributions and concentrations for each stream in the pilot plant—given the water recoveries and froth recoveries of that system—then it is considered validated. The resulting validated model is then used to simulate the industrial plant, with corrections made for froth depth, air rate, and mechanical constraints.

Pilot plants are also useful for producing large samples of concentrate and tails, which is often necessary for downstream process test work and design.

Test Interpretation

The discussion on test interpretation is limited to kinetics tests, as these are the only tests that are capable of providing the first-order rate information required by phenomenological models. In general, there are three steps to interpreting a batch kinetics test and extracting the mineral-specific rate constants and maximum recoveries. They are listed as follows:

1. Ensure that the assays and the mass and water inputs and outputs are balanced using an accepted mass balancing method, of which there are many.
2. Convert the balanced assays into balanced minerals, calculating the assay of gangue by difference. The gangue assay can be checked by X-ray diffraction or other quantitative means if required.
3. Calculate the kinetics parameters for each mineral, including gangue, using a least-squares fitting method applied to Equations 1 and 2a.

In cases where one element can contribute to more than one mineral, it may be advantageous to model kinetics on a mineral basis rather than on a chemical assay basis, as different mineral phases may exhibit different behaviors regardless of their elemental constituents. A secondary advantage to modeling on a mineral basis is that by converting all elemental assays into their respective minerals, the mass fraction of gangue can be calculated and modeled as a separate mineral class with its own specific gravity, allowing for the calculation of specific gravity in the product streams. The disadvantages include higher costs associated with mineralogical determinations, and the additional complexity required to convert from assays to minerals and back again.

The fitting of Equation 1 to batch kinetic data is not straightforward, and the following considerations may be of assistance:

- It is helpful to treat each time interval as a separate *reactor*, with the tails from the first concentrate interval being the same as the feed to the second interval, and so on until the final test tails are reached. This allows for the use of separate froth recoveries for each stage (Vera et al. 2002; P.A. Amelunxen et al. 2014).
- Often it is preferable to fit the curve of entrainment versus particle size, rather than just the mean entrainment (for all particle size classes). This allows for modeling different size distributions for each concentrate period, such as when accounting for removal of fines because of entrainment in prior periods.
- It is important to always measure the percent solids of the concentrates; otherwise the cumulative water recovery cannot be determined, and therefore entrainment cannot be calculated or fitted.
- Unless using a phenomenological model for R_f and R_e (P. Amelunxen et al. 2018), the kinetics parameters for all minerals should be fitted simultaneously, as the test will not generate enough degrees of freedom to calculate them independently.
- Most engineers assume that the froth recovery in a batch test is 100%, but recent work has shown that this is decidedly not the case. The froth recovery is strongly dependent on the scraping rate, and even with continuous scraping (one complete scrape every 1–3 seconds), the R_f is significantly below 100% (P.A. Amelunxen et al. 2014). It is also important to validate the model through detailed plant surveys to correct for differences in froth recovery between the laboratory and the plant.

SIMULATORS

Steady-State Simulators

Most equations, such as the preceding, for industrial flotation circuit simulation are based on the assumption that the circuit being simulated is operating in dynamic equilibrium and that the mass, mineral, and energy flow into and out of the system are at steady state. This greatly simplifies the mathematical and computational handling of the calculation.

Steady-state flotation simulators are simply ensembles of flotation cell models. Each model is configured with the properties of the industrial cell (size, depth, wash water, etc.), which are then connected together in such a way that a circuit is formed. Several cells form a bank, several banks form a row, several rows form a stage, and several stages form a circuit. The actual method of connecting the various streams may require some ancillary models—splitters, combiners, and so forth. These can be subdivided as well. Splitters can be mass-based (header boxes) or size-based (hydrocyclone, screens, other classifiers). Most flotation circuit models involve circulating loads and iterative calculations, and, therefore, require some kind of simulation engine, which is the logic that drives the model convergence. This logic can be programmed in any language; some examples of platforms that have been used for flotation models include Fortran, Basic, Excel, Matlab, C++, C#, and R.

In cases where reagents are added midcircuit, it is necessary to account for this effect. Commonly, kinetics tests are performed using slurry collected from the plant at any point in which the surface chemistry or mineral liberation has been modified. For circuit design, it is often necessary to create an artificial sample for subsequent kinetics tests (P.A. Amelunxen et al. 2013).

To make it all work, two basic features are absolutely required and a few are recommended. The must-have features of a phenomenological steady-state flotation simulator are as follows:

- **The ability to track kinetics.** The R_∞ and k parameters that are measured in the laboratory, whether on an averaged or size-by-size basis, are only valid for the first cell in a rougher bank. The simple action of removing floatable material via the concentrate of the first cell reduces the kinetics of the feed to the second cell. The same must be true in reverse: When combining streams with different kinetics, the resultant kinetics must reflect that of the blend of streams. Models can account for this in various ways; some use the concept of differential kinetics (P.A. Amelunxen and Amelunxen 2009), and others track the floatability component of each mineral class separately (Runge 2007).
- **The ability to predict concentrate grades.** The phenomenological model must accurately predict stage recovery and concentrate grades for all minerals including gangue. This allows for the calculation of mass flow, bulk specific gravity, and slurry percent solids for each stream. These parameters are essential for calculating the residence times of subsequent cells and stages, and ultimately for circulating load and concentrate grade determinations.

In addition to the preceding, the following features should be given serious consideration. They are not absolutely critical, because they can be done manually (outside the framework of the simulation tool), even if they require more time to do so:

- **The ability to consider mechanical constraints.** Flotation machines have mechanical limits as well as metallurgical limits. The flotation model must consider such things as lip loading constraints, carrying rate limits, launder, pump and pipeline constraints, and froth crowding effects.
- **The ability to mimic some basic human logic.** When using a geometallurgical approach to plant design, it is common to change the process configuration assumptions based on the ore type being processed during any given period in the mine plan. For example, it is often desirable to change the regrind circuit specific energy based on the mineralogy and liberation characteristics of the ore type or to bypass a cleaner stage when processing higher grade ores. Modern flotation circuit design is an economic optimization exercise across a mine sequence plan. To avoid suboptimal outcomes (or time-consuming optimization tuning), the simulator should be flexible enough to incorporate some basic human logic to account for this.

Some other features are often found with flotation simulators:

- **A comminution circuit model** allows for simultaneous optimization of the comminution and flotation circuits as part of a more holistic approach to concentrator design.
- **An economic model** includes smelter treatment and refining charges, contaminant penalty charges, concentrate

freight costs, reagent consumptions and costs, grinding media consumption and costs, energy consumption and costs, capital costs, and other key economic parameters that are needed to evaluate a simulation based on economics, rather than mineral recoveries and product qualities.
- **An optimization engine** allows for searching through a range of possible operating conditions and configurations and identifying one or more economically optimal configurations that meet any required objectives. These can range from simple bisectional search routines to complex partial derivative and/or simplex algorithms.
- **Geometallurgical modeling functionality** includes the ability to simulate circuit performance across an array of ore types, head grades, grind sizes, and other conditions. This can be done on an individual sample basis, on a time period basis, or on a block model basis.
- **A Monte Carlo simulation** component allows for error propagation analysis, confidence estimation, risk evaluation, and other quality assurance and control studies.
- **A configurable process design output** supports rapid generation of key design documents, including process flow diagrams, process design criteria, equipment lists, and mass balance tables.

Dynamic Simulators

Up to this point, discussion on flotation models has focused on steady-state phenomenological models. Recently, there has been an increasing use of dynamic and physics-based flotation modeling software and methods. In practical applications, such as for the design and engineering of new concentrators, the use of these has been limited to the determination of required surge capacities, launder and pipe sizing, and process control requirements. In the future, it is possible that this class of model will become the more favored approach for process optimization and plant design (Herbst et al. 2005).

Limitations of Phenomenological Models and Simulators

All of the phenomenological models and simulators just discussed are merely mathematical approximations of a complex, dynamic process. Because they are only simplifications, they are neither capable of nor intended to completely and correctly describe all of the mass and energy transfer phenomena that occur in a flotation machine or circuit. Some of these simplifications are necessary because of inadequate scientific understanding, while others may stem from the inability to economically measure certain system parameters under the practical or economic constraints of industrial metallurgical laboratories or test programs. Regardless of the reason, it is the responsibility of the practitioner to understand, if not manage, the trade-off between fidelity and pragmatism. Some of the known limitations include the following:

- Equation 1 does not consider the energy input to the system, one of the most important variables and the key component of the operating cost of the equipment.
- The rate constant is the sum of attachment and detachment rates; most models do not allow for separate submodels of detachment and attachment rates. As the energy input changes, the attachment and detachment rates increase or decrease at different rates, changing the overall rate constant.
- Most phenomenological models do not include an interface zone.
- Most phenomenological models do not consider atmospheric pressure.

Researchers are working on addressing these shortcomings.

- Some have proposed amendments to Equation 4 to incorporate the role of energy intensity and turbulence (Sherrell 2004; Tabosa et al. 2011; Amini et al. 2015).
- In some works, the rate constant has indeed been expressed as the sum of the attachment and detachment rates, although quantifying them has been challenging. See, for example, Deglon et al. (1999) and Sherrell (2004).
- Some have proposed that a third compartment—the interface—be added to Equation 3, as proposed by Seaman et al. (2004).
- Work is progressing on characterizing the role of barometric pressure in froth recovery (Young 2007).

It should also be remembered that having a model is not the same thing as being able to scale up from the laboratory to the plant. Just because one has developed an equation or set of equations that can describe the flotation system to a desired level of fidelity does not mean that one can measure the required inputs at the laboratory scale, or use that equation to predict the behavior of a different system. By far, the most common commercially used laboratory flotation cell is the 1940s-era Denver benchtop flotation cell, and most laboratory tests can only characterize the inputs and outputs of the system—feed grades, particle size, water and reagent concentrations, air flow, and sometimes energy input. Many of the inputs and almost all of the intrinsic parameters remain unmeasured. These include entrainment, gas dispersion, attachment and detachment rate, drainage rate, and air recovery (or froth coalescence).

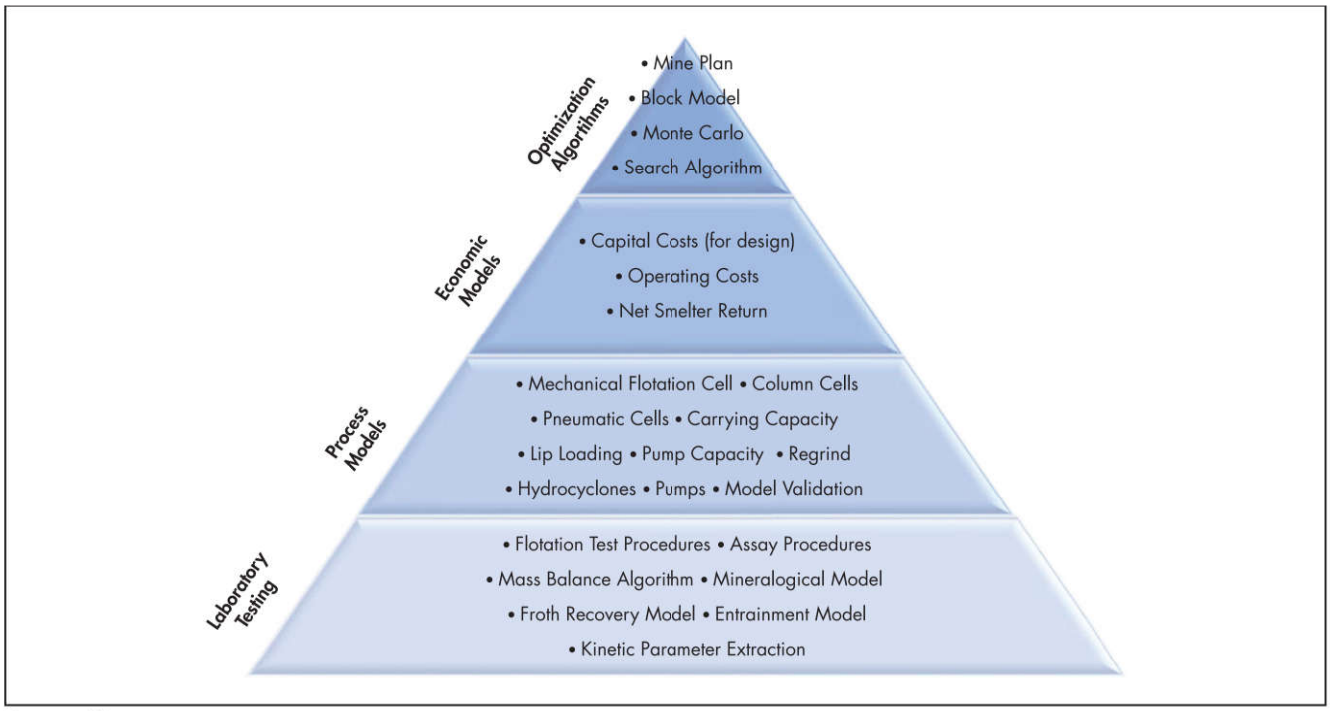
There is another complication associated with the flotation modeling methods described earlier. Many of the unmeasured or unmeasurable parameters change significantly from laboratory to plant. This introduces a very significant complexity into the scale-up process, because now not only must they be characterized (or, more likely, a value assumed for them) in the laboratory, but how they change between the laboratory scale and the plant scale must be understood. To illustrate, Table 1 lists some of the important system parameters, how they are thought to change between laboratory scale and plant scale, and what the effect on the flotation performance may be.

It is precisely the nature of, and differences between, these assumptions that require that any flotation model or simulator be validated. The process of model validation is straightforward; it involves performing a sampling campaign around a target flotation circuit to determine the mineral and mass flow rates of each stream. Kinetics tests are performed on the various nodes (as a rule, a flotation test should be performed on any node in which the surface chemistry has been modified, such as by regrinding, a change in pH or reagents, or a significant change in the slurry density).

The kinetics parameters are then extracted and input into the simulator. The simulator is run, and the froth recovery, water recovery, and degree of entrainment are adjusted

Table 1 Difference in flotation system parameters between plant and laboratory

Parameter	Laboratory	Plant	Effect
Feed solids			
P100	<425 μm	<1,180 μm	Lower R_{max} in the plant
P80	No change	No change	No change
P50/P80 ratio	0.4–0.6	0.25–0.5	Higher R_{max}, faster kinetics in laboratory
Time in grinding mill	15–70 minutes	3–15 minutes	Depends on media type; generally more pyrite passivation in plant
Feed flow	Batch (no flow)	Continuous	Plug-flow-reactor model in laboratory; continuous stirred-tank reactor model in plant
Feed and makeup water			
Water type	Usually tap or sea water	Reclaimed process water	Recycled process water contains residual frother, reagents. Very different frothing characteristics in plant
Water recovery	10%–65%	1%–10%	Higher entrainment rates in the laboratory
Water flow	Semi-continuous	Continuous	Higher water recovery in laboratory, higher oxidation potential in laboratory
Gas flow			
J_g	0.4–0.7	0.5–2.2	Slower kinetics in laboratory, lower R_f in laboratory
S_b	25–40	50–180	Slower kinetics in laboratory, lower R_f in laboratory
D_b distribution	Narrower	Wider	Faster kinetics in laboratory
Gas flow	Continuous	Continuous	No change
Froth			
Froth height	1–2 cm	4–150 cm	Faster kinetics, more entrainment in laboratory
Froth residence time	2–20 seconds	10–500 seconds	Faster kinetics, more entrainment in laboratory
Pulp–froth interface	Quiescent	Turbulent	Lower entrainment in laboratory
Pressure	Depends	Depends	Depends on elevation, cell type
Energy input	Higher	Lower	Higher attachment, detachment rates in laboratory
Degree of entrainment	Lower	Higher	Lower laboratory ENT for a given particle size, may be offset by higher R_w



Courtesy of Aminpro

Figure 5 Simulation and scale-up methodology

until the model agrees with the plant balance. If the model is capable of predicting the solids flow rates for each stream, the mineral grades of each stream, and the mineral recoveries of each stage without the use of correction factors or tuning parameters, then the model is valid. Many examples of model validation studies can be found in the literature (Coleman et al. 2007; Dobby and Savassi 2005; P.A. Amelunxen and Amelunxen 2009).

This is a good place to comment that models do not always agree with the plant performance. More generally, flotation models are tools for *improving* our understanding of a complex, nonlinear process. They are not a *substitute* for understanding. In the authors' experience, it is precisely when the model fails to predict the plant performance that the most interesting lessons can be learned.

SUMMARY

The phenomenological modeling of flotation circuits can be thought of as a pyramid (Figure 5). At the base of the pyramid, forming the foundation, is the laboratory testing methodology, including the test procedures and equipment that are used for the work, the kind and quality of the data collected, and the treatment of that data to extract useful information for subsequent modeling and simulation.

The modeling and simulation step sits on top of the foundation of laboratory testing. This contains the basic mathematical and software framework of the model, including the dual-compartment model, lip loading and carrying capacity constraints, pumping constraints, and submodels for the various types of flotation cells—all linked together within the framework of a simulation platform.

Once the basic mathematical framework is complete, it is capable of basic simulation work. Whereas some simulators only contain these two levels, others are equipped with additional economic models and optimization algorithms, as shown in Figure 5, the pyramid diagram.

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PART 8

Solid and Liquid Separation